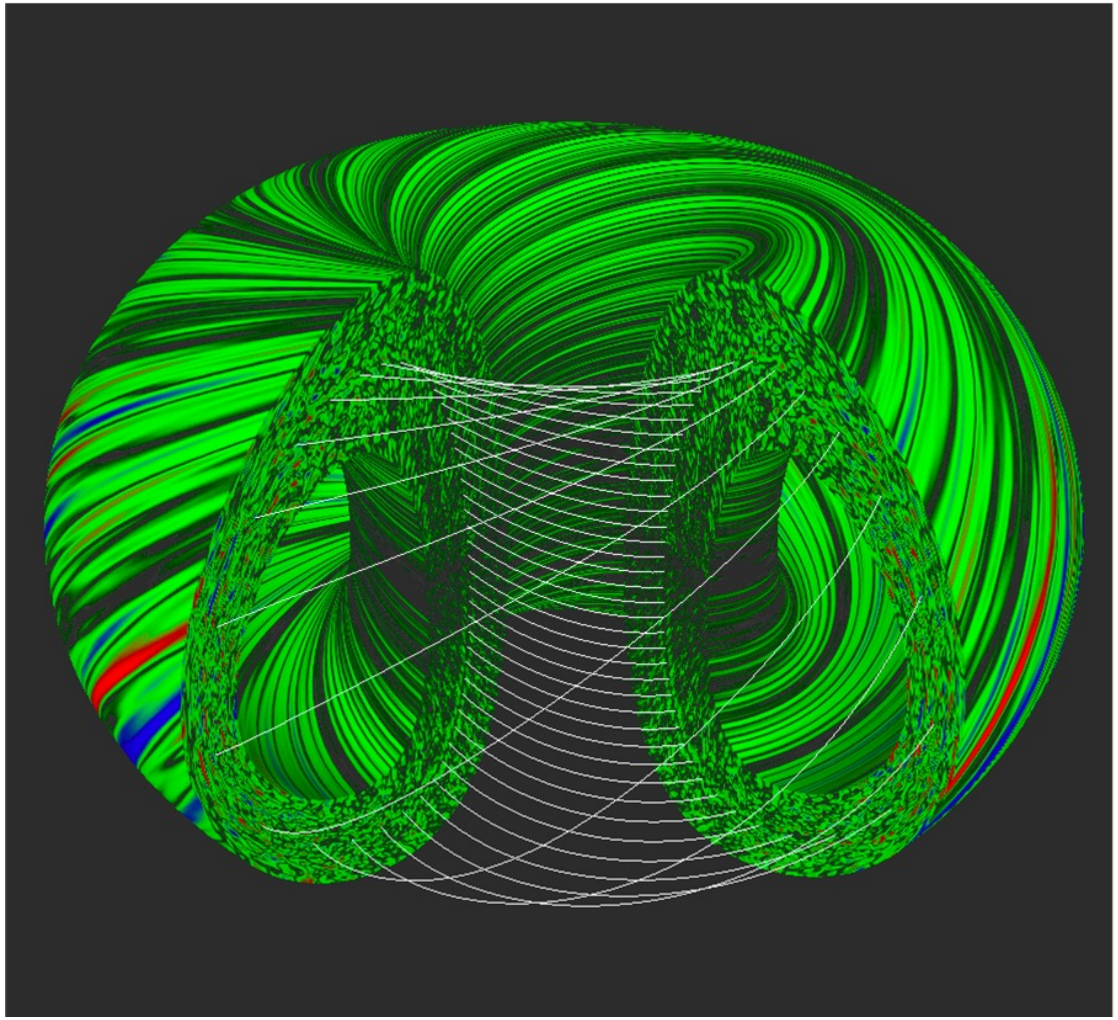
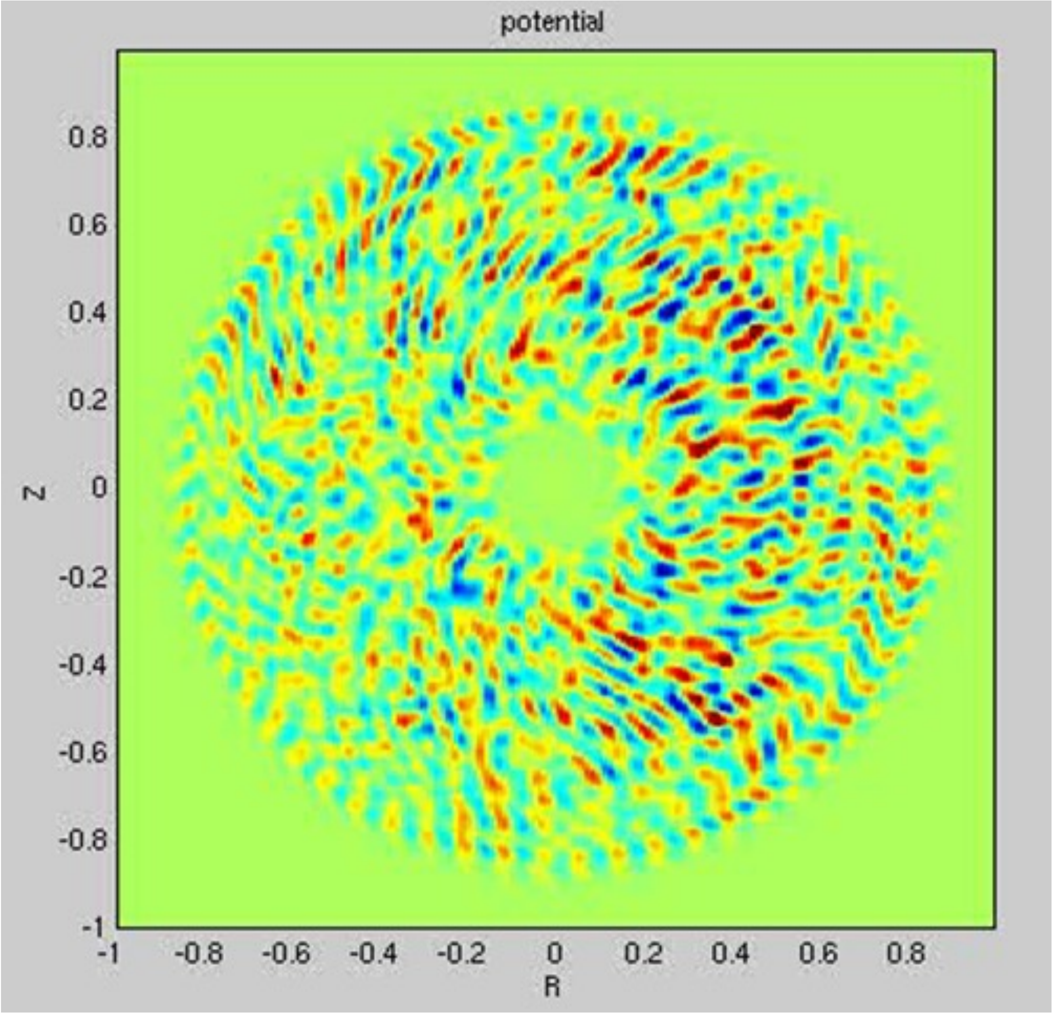
Introduction to gyrokinetics –part 2

Josefine H.E. Proll
– with great thanks to my PhD students Maikel Morren and Paul Mulholland

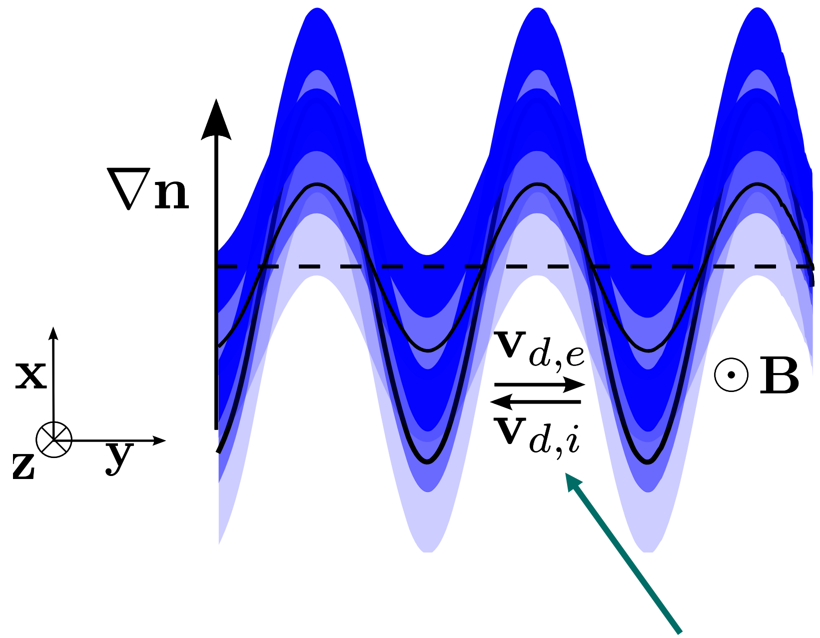
This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

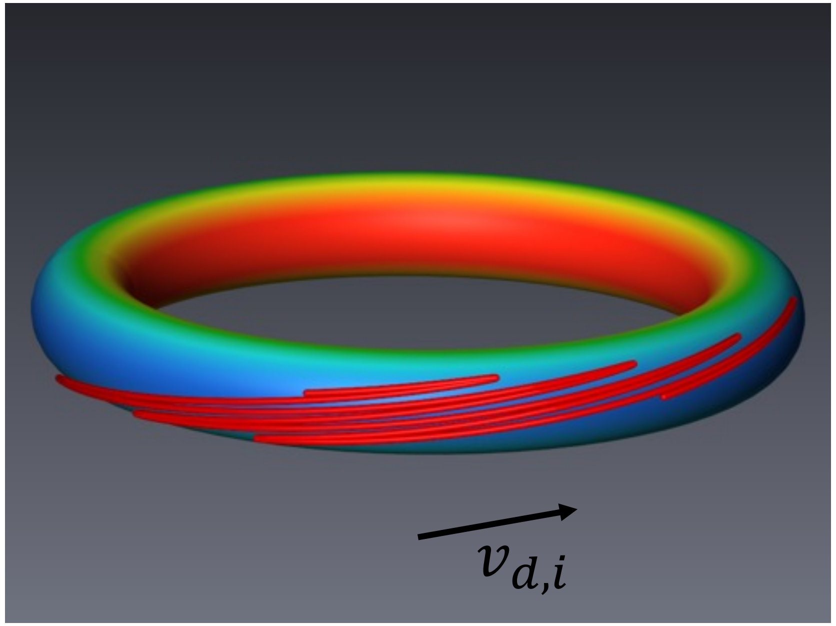
How do we end up with turbulence?





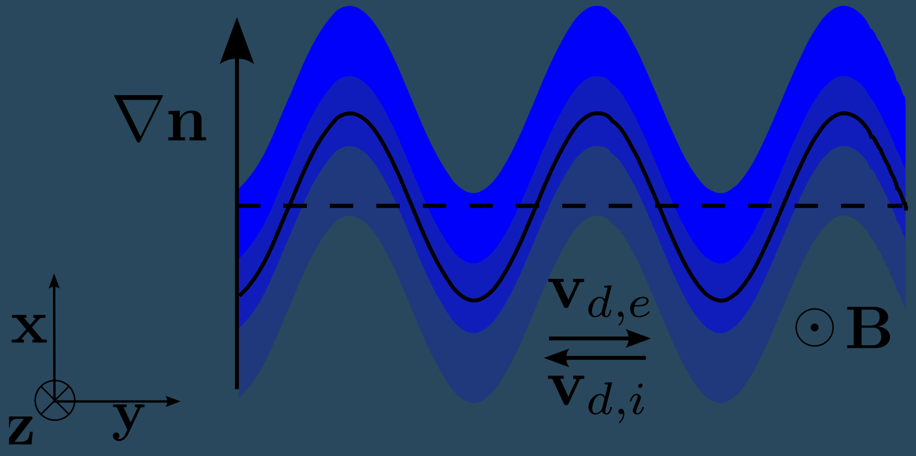
Turbulence comes from perturbations that grow (i.e. instabilities), which then interact





Geometry of magnetic field

Introduction to gyrokinetics
- the most important instabilities



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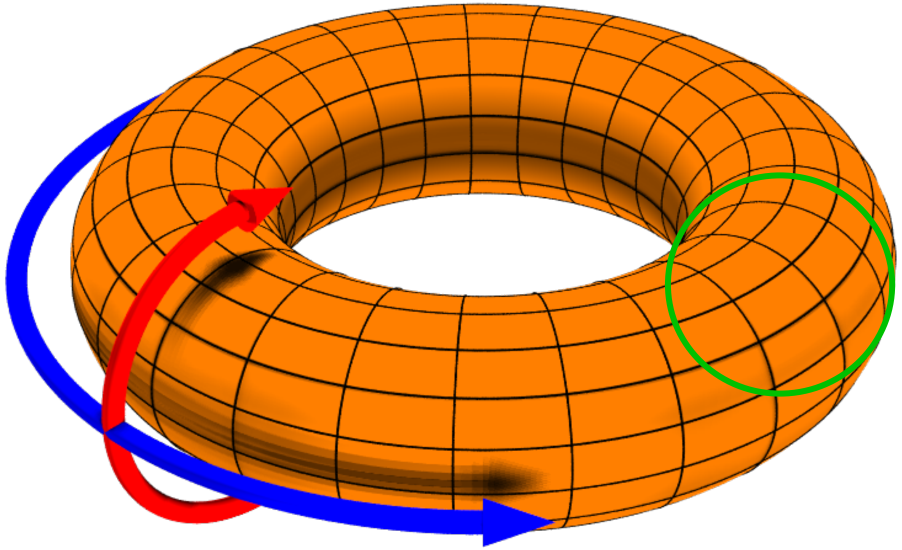
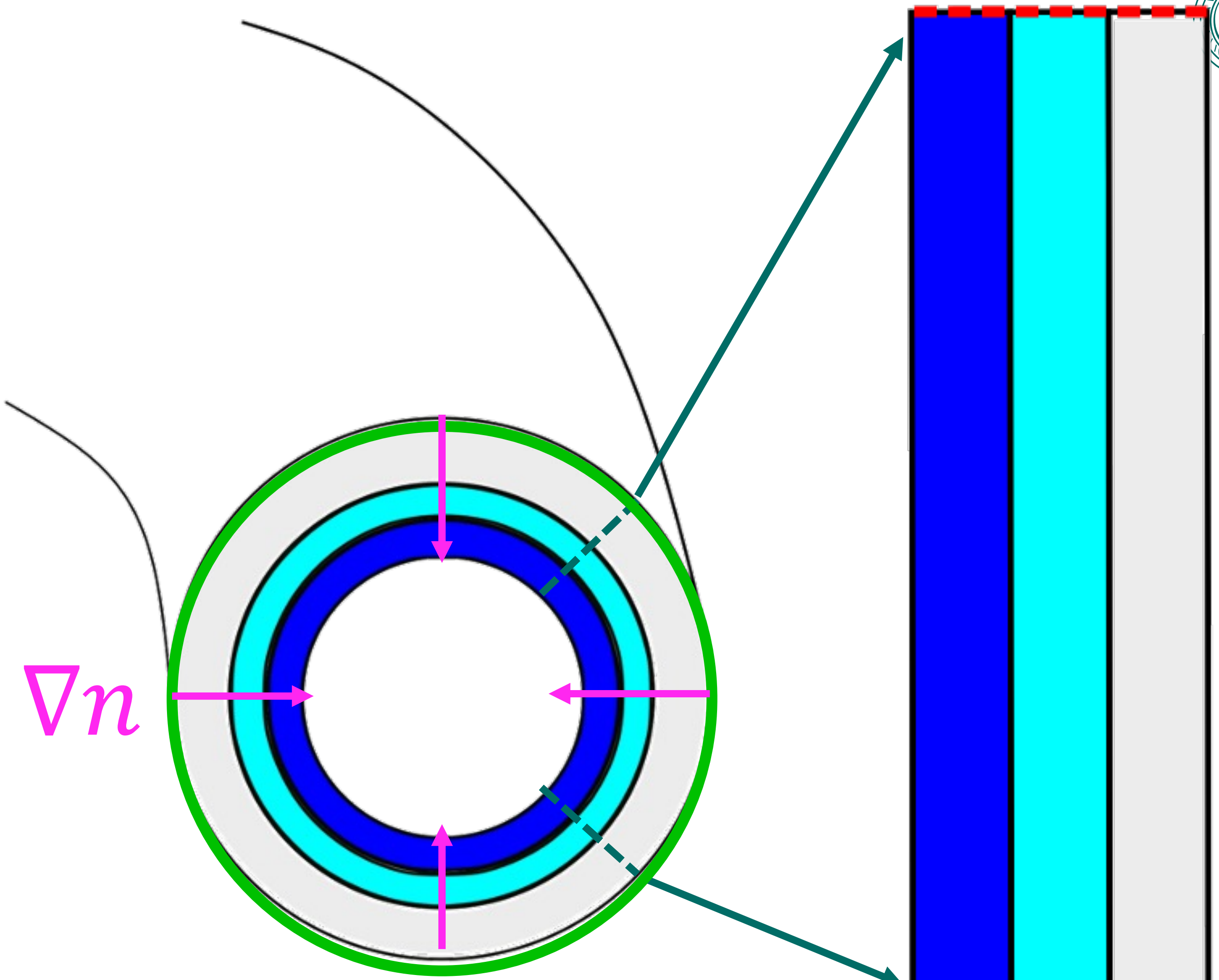
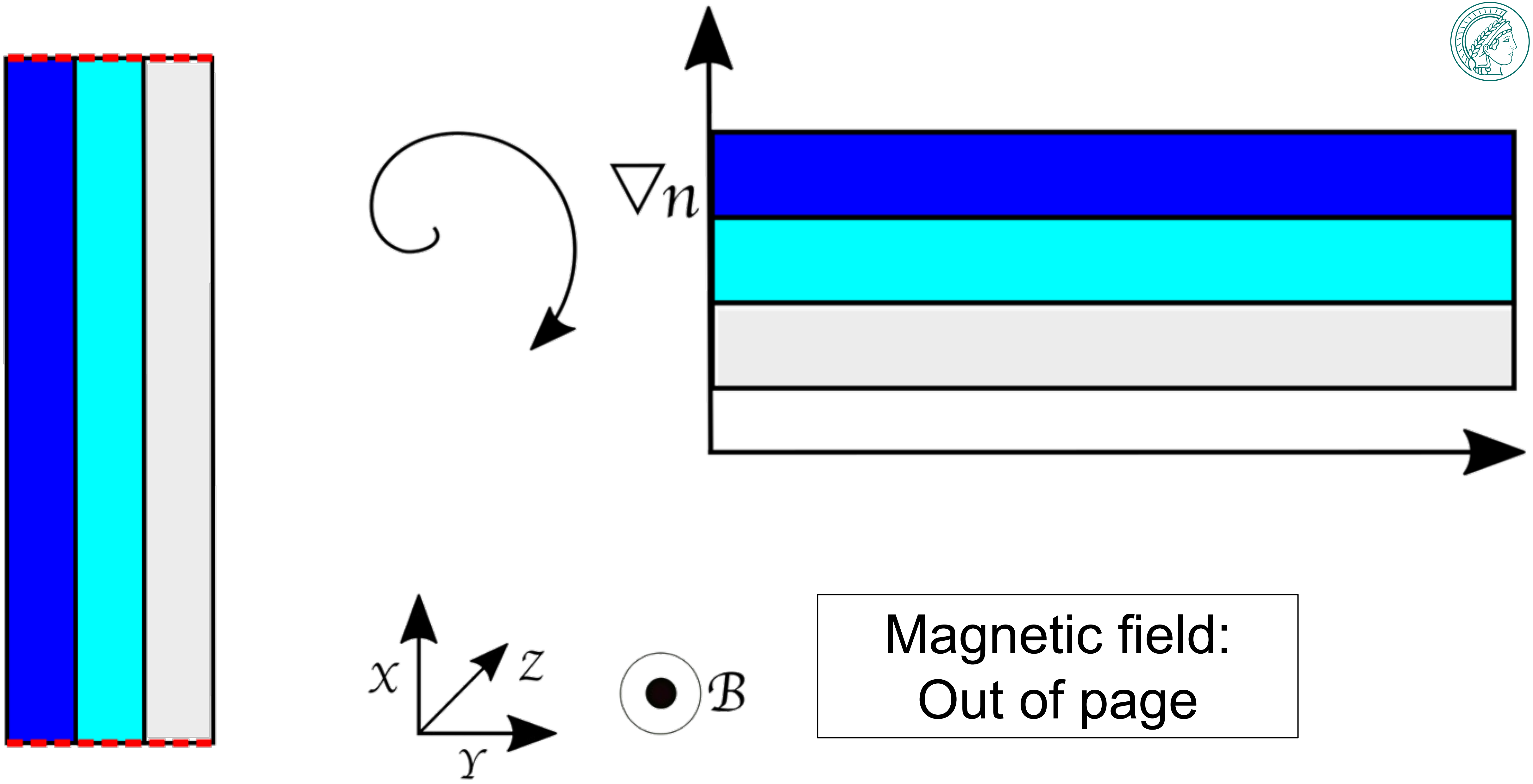


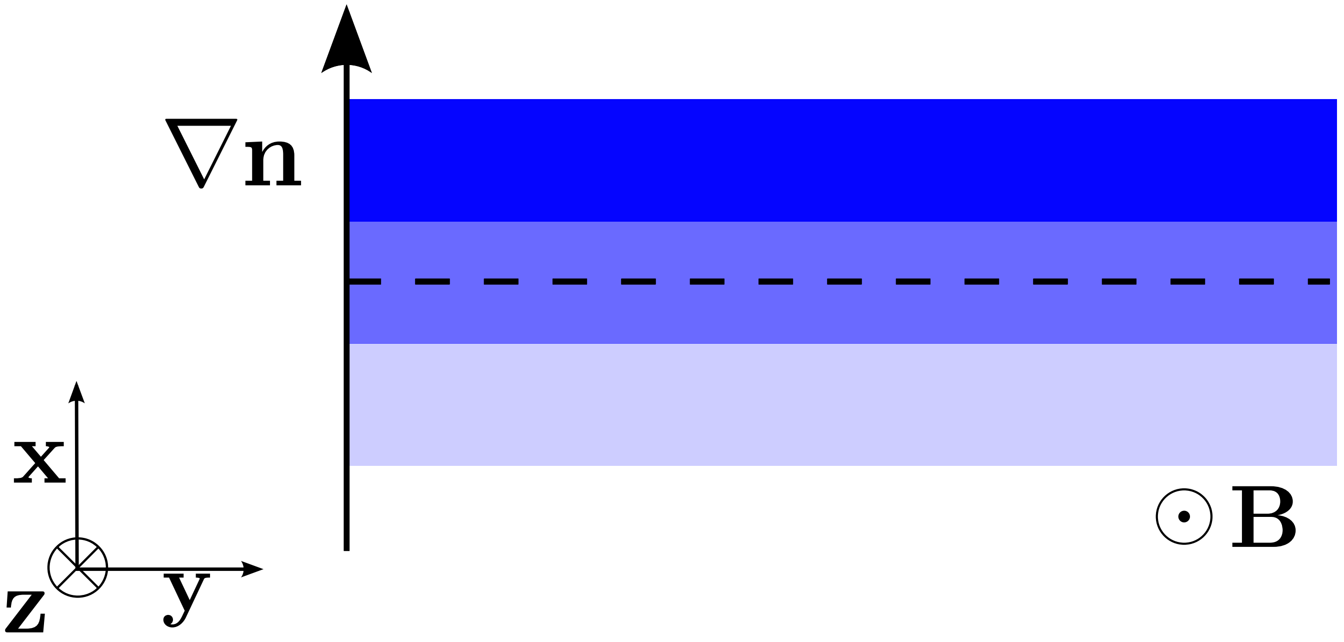


Thanks to Paul Mulholland for this animation!

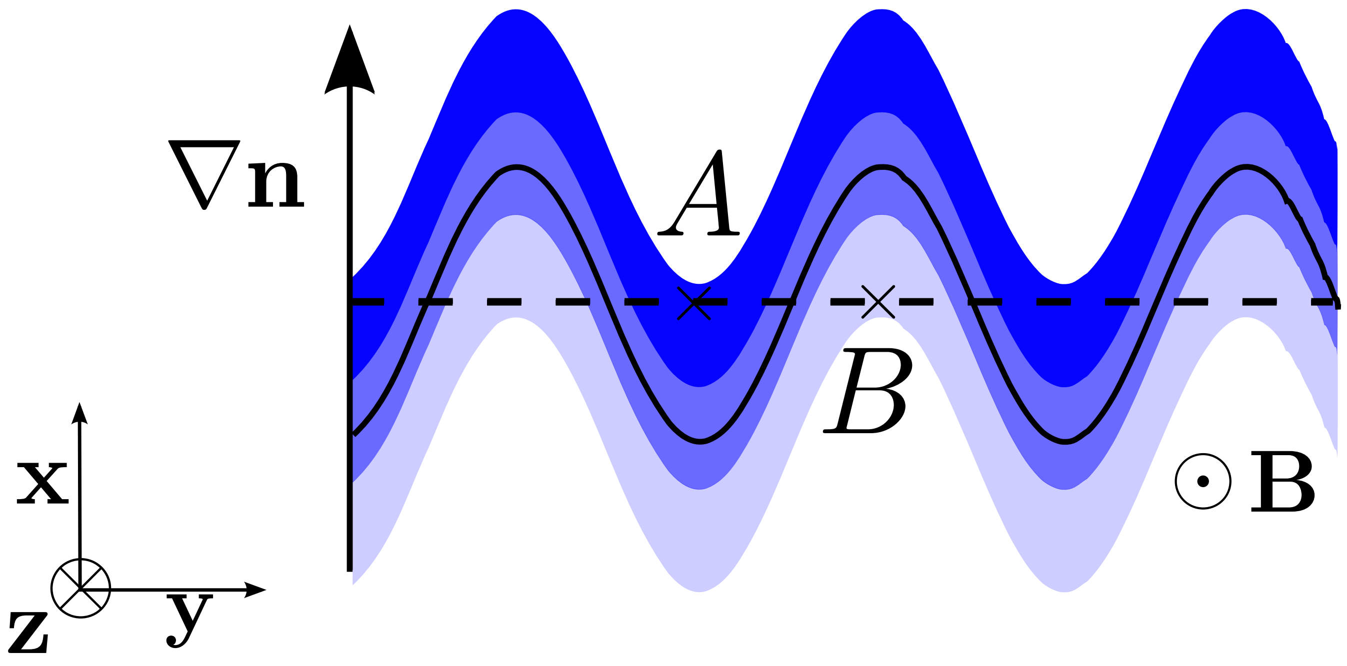


∇n
X
Z
Y
B
Magnetic field:
Out of page

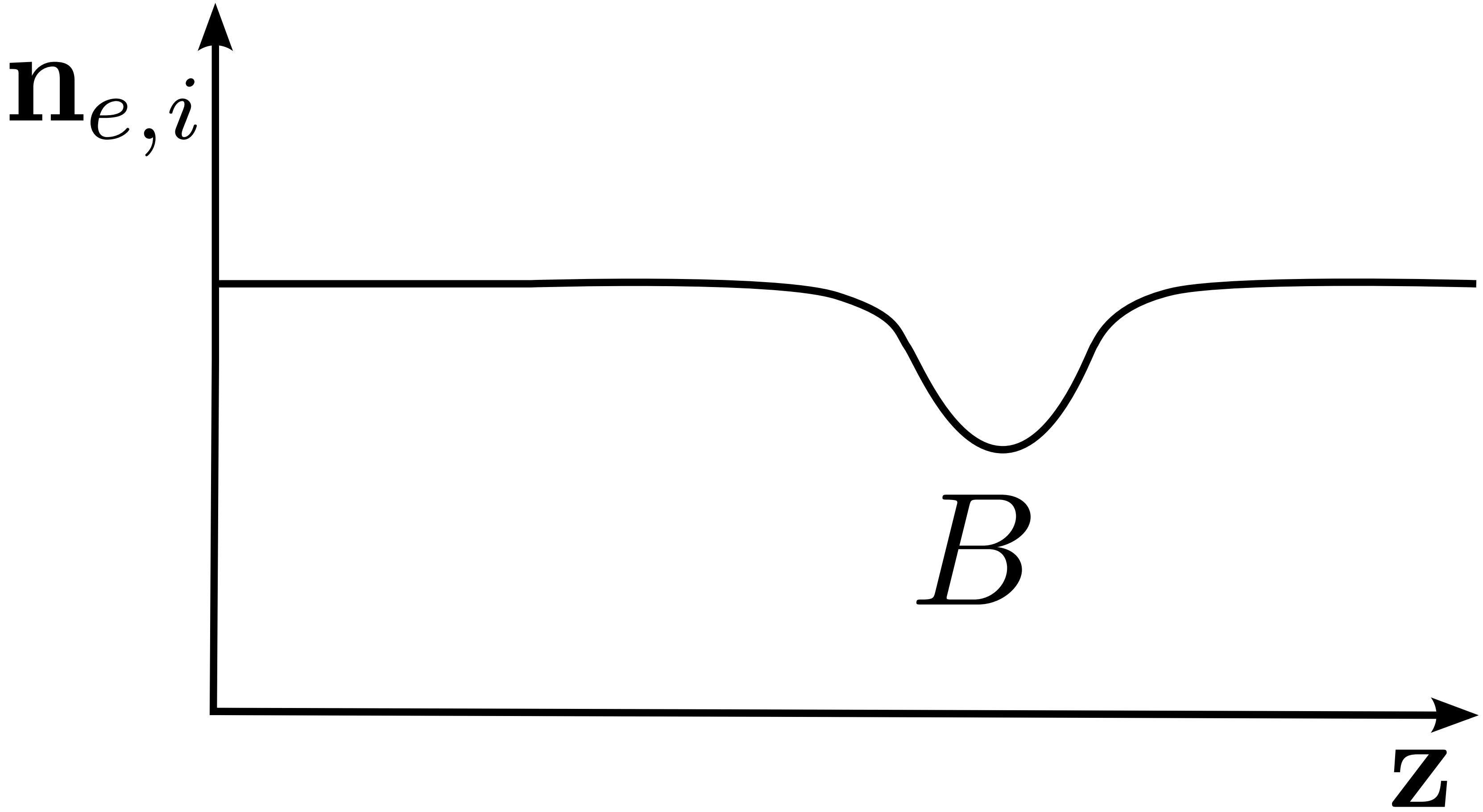
Drift waves



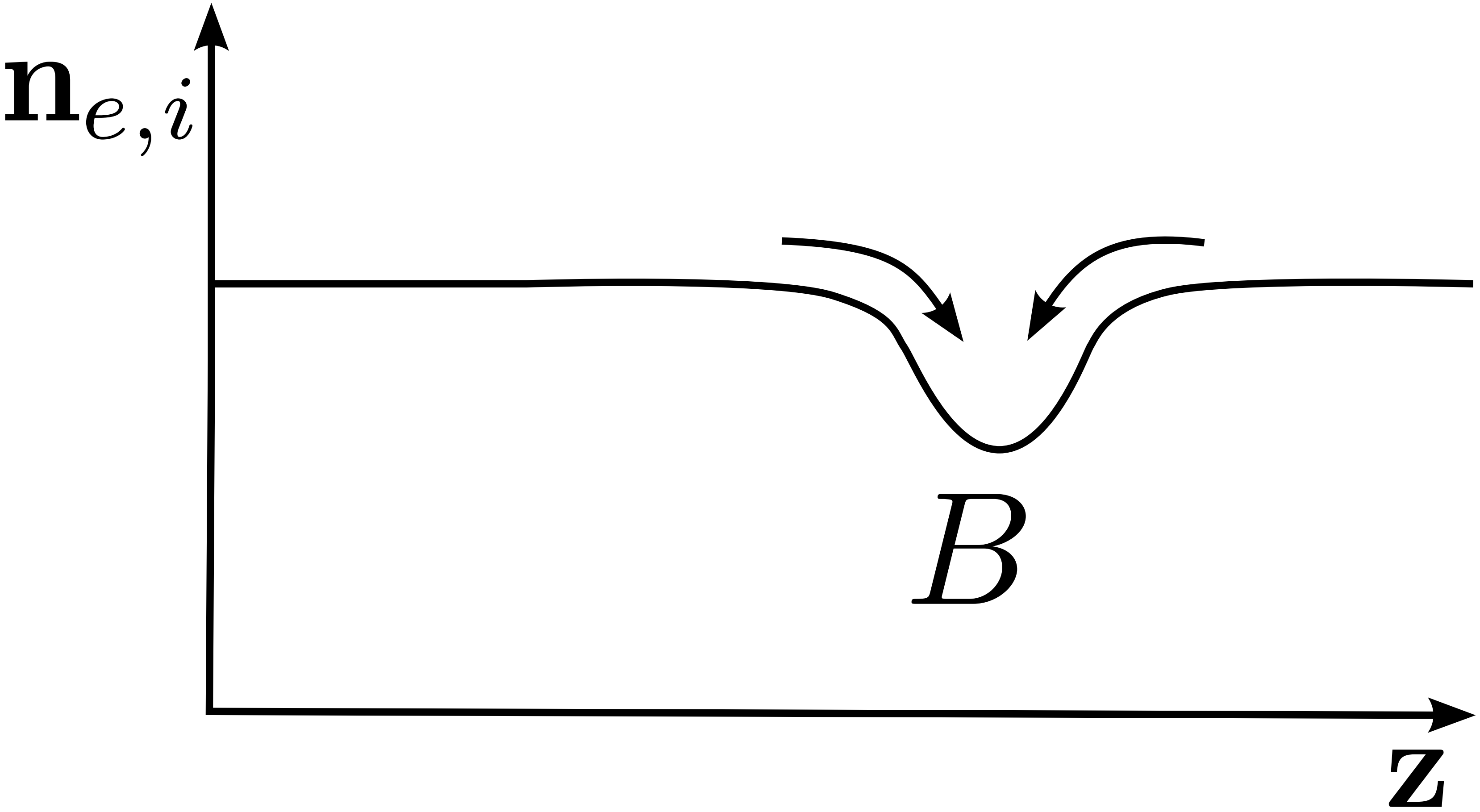
Drift waves



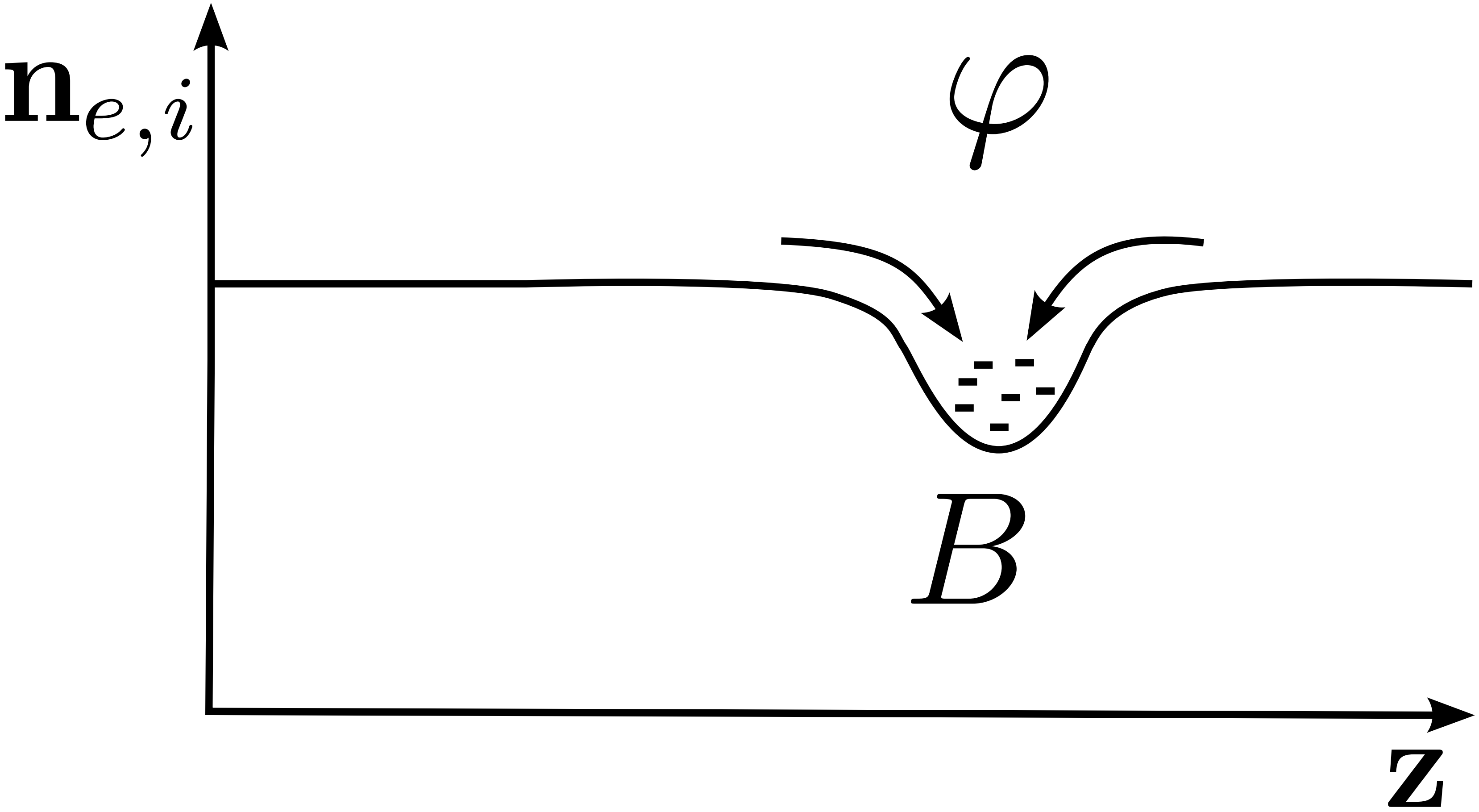
Drift waves – adiabatic response of the electrons



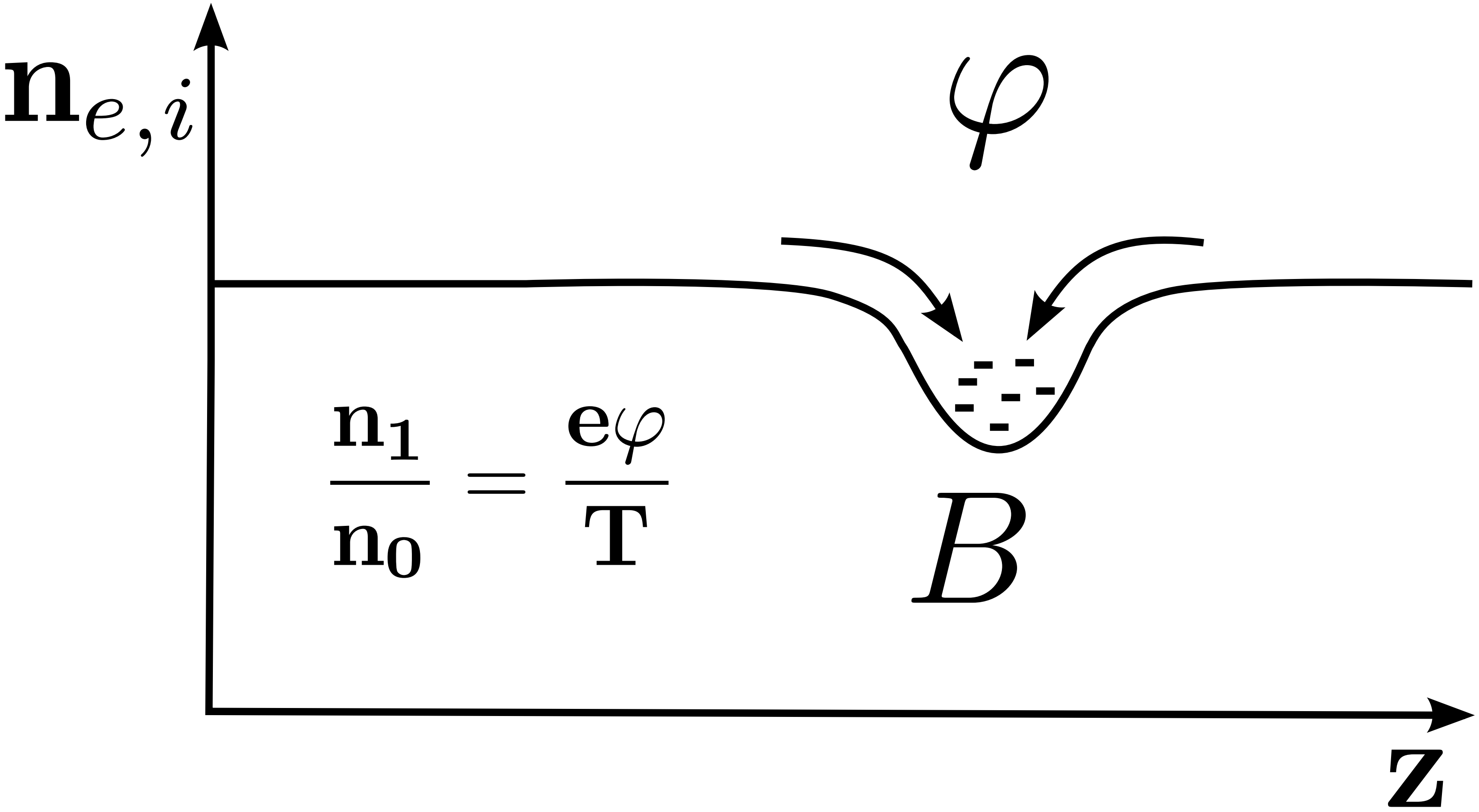
Drift waves – adiabatic response of the electrons



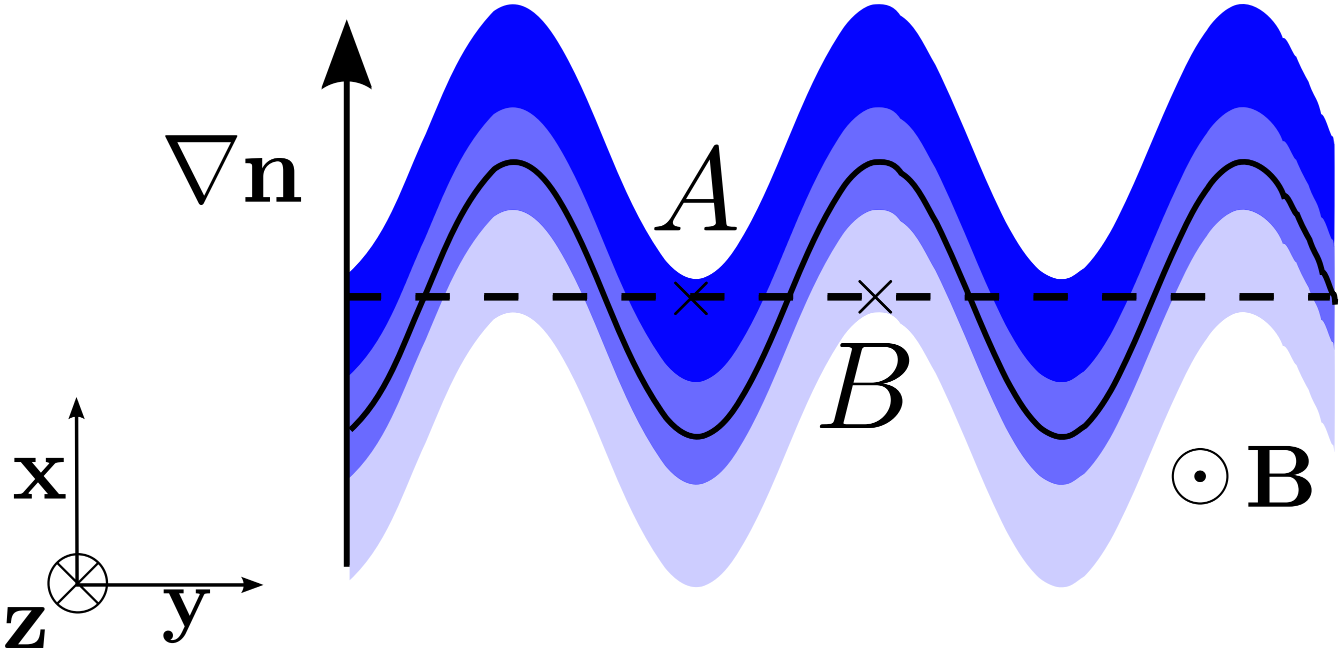
Drift waves – adiabatic response of the electrons



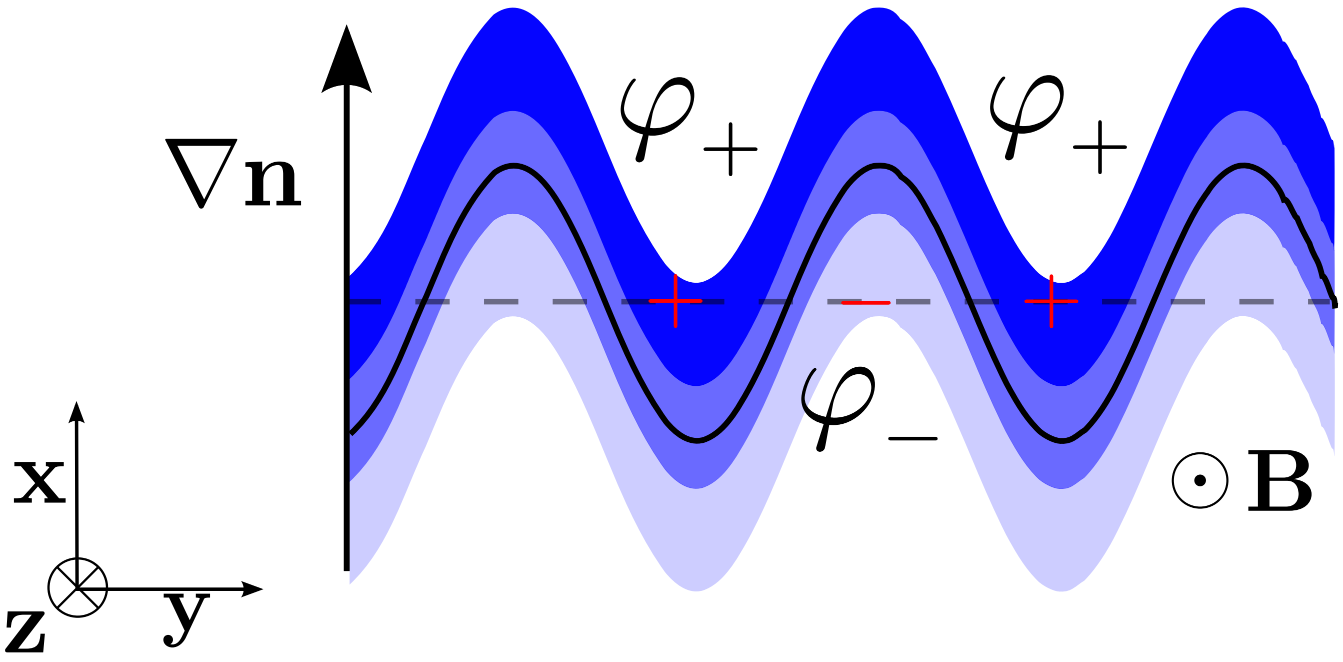
Drift waves – adiabatic response of the electrons



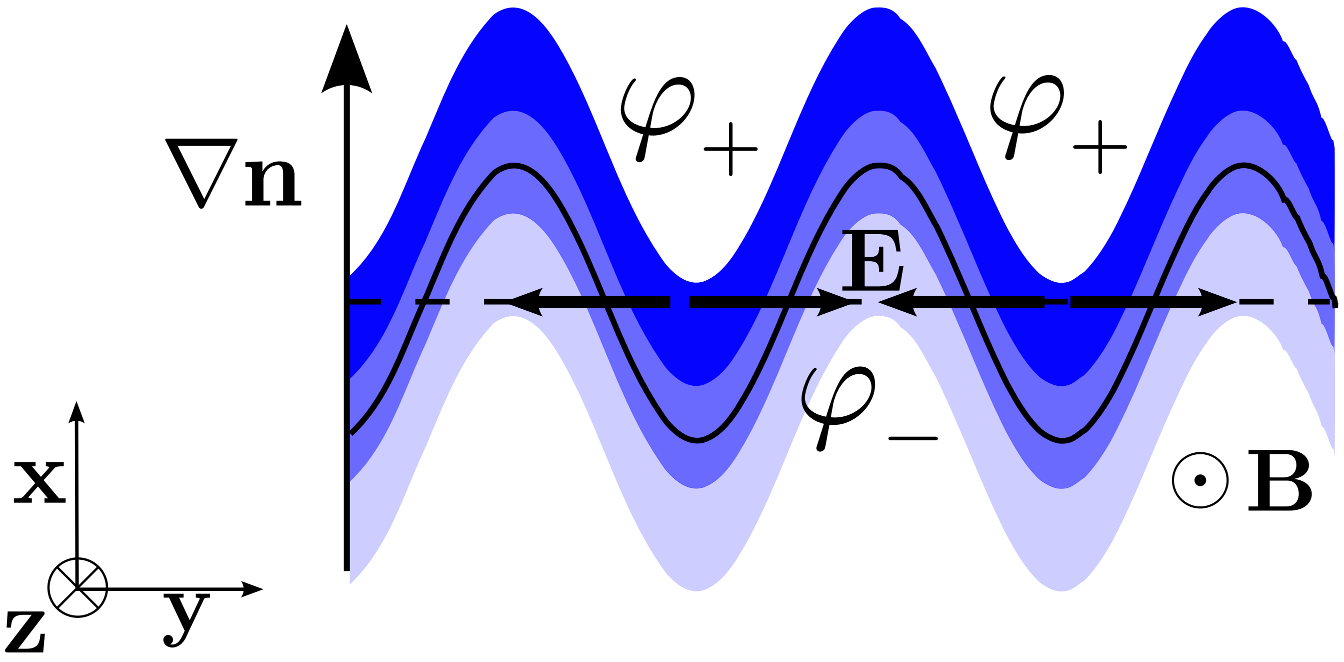
Drift waves



Drift waves

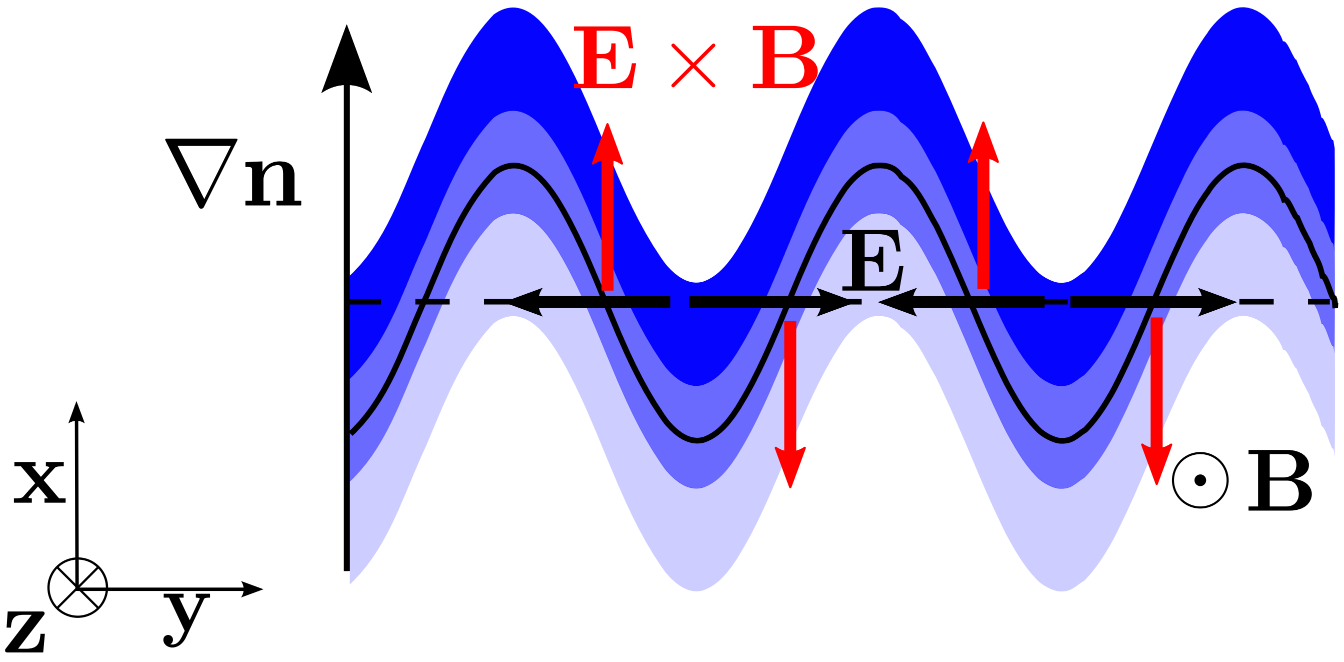


Drift waves

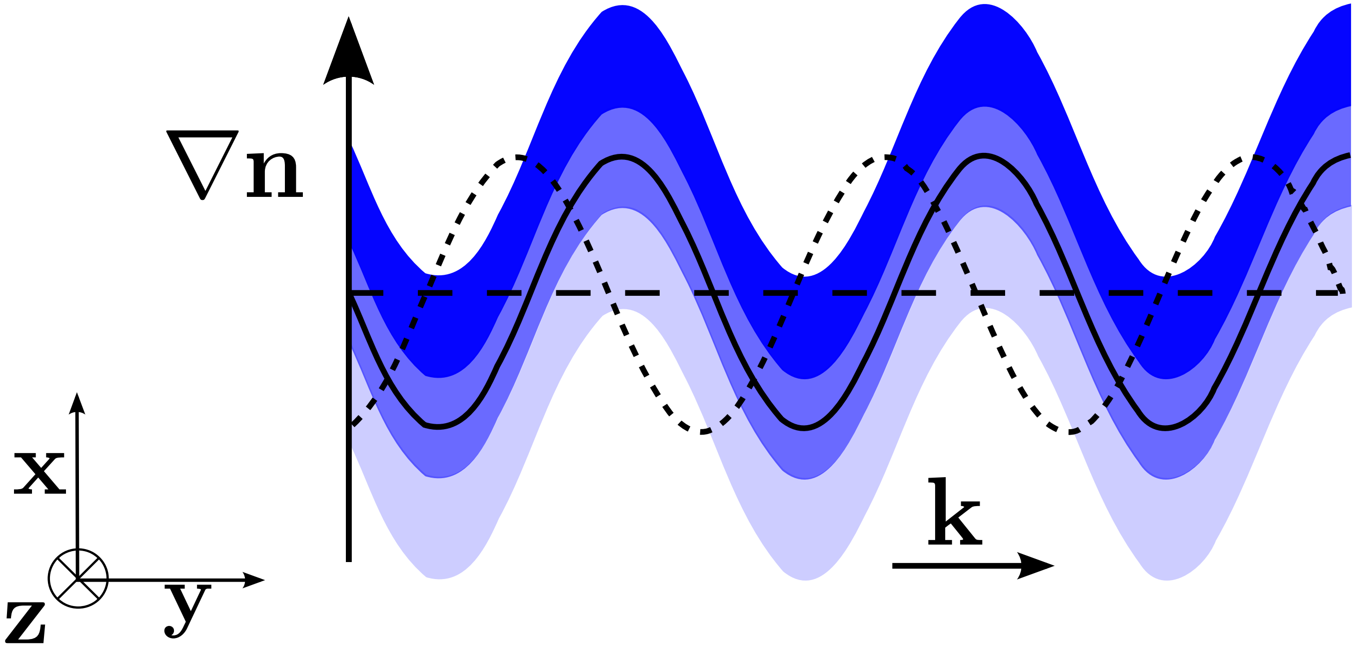


$\nabla \mathbf{n}$
φ_+
φ_+
$\mathbf{E}$
φ_-
$\odot \mathbf{B}$
$\mathbf{x}$
$\mathbf{y}$
$\mathbf{z}$

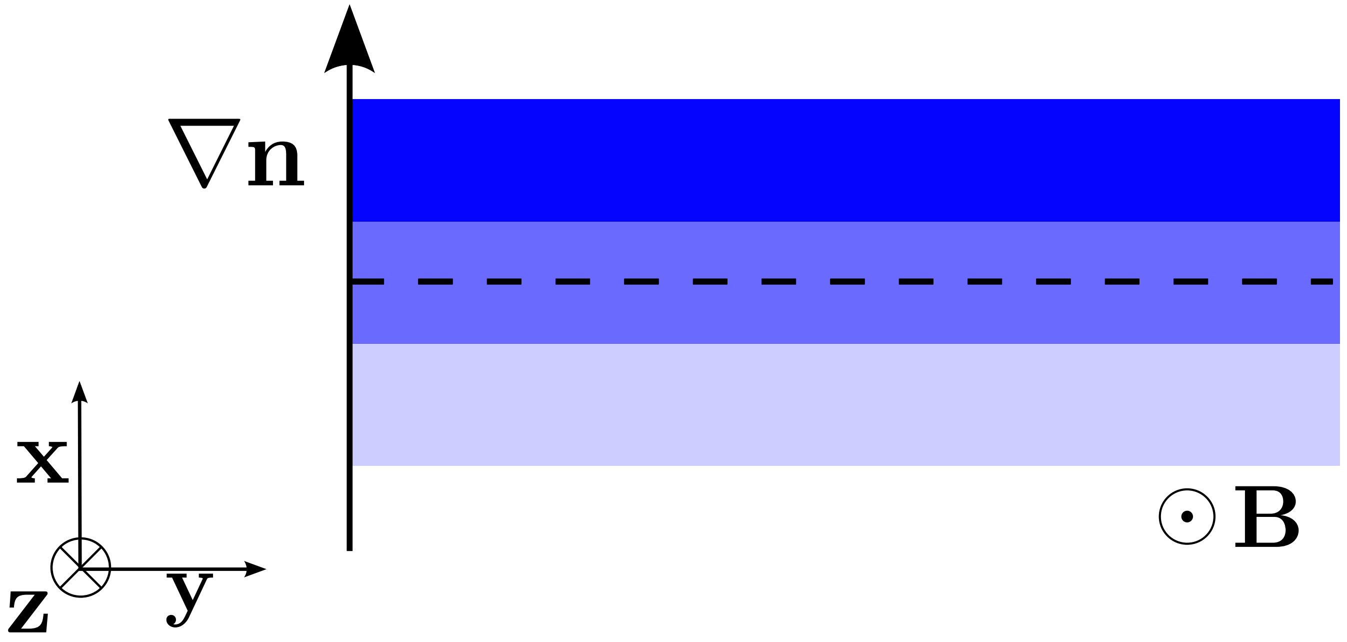
Drift waves



Drift waves



Trapped-particle mode



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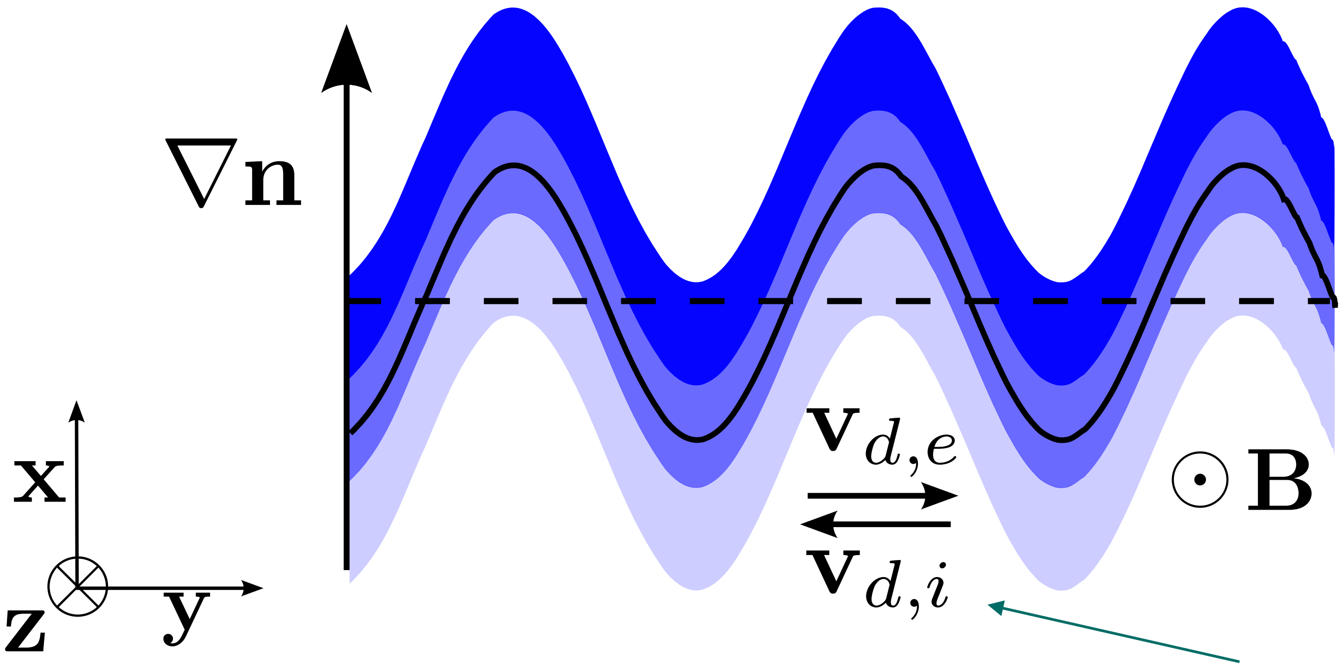
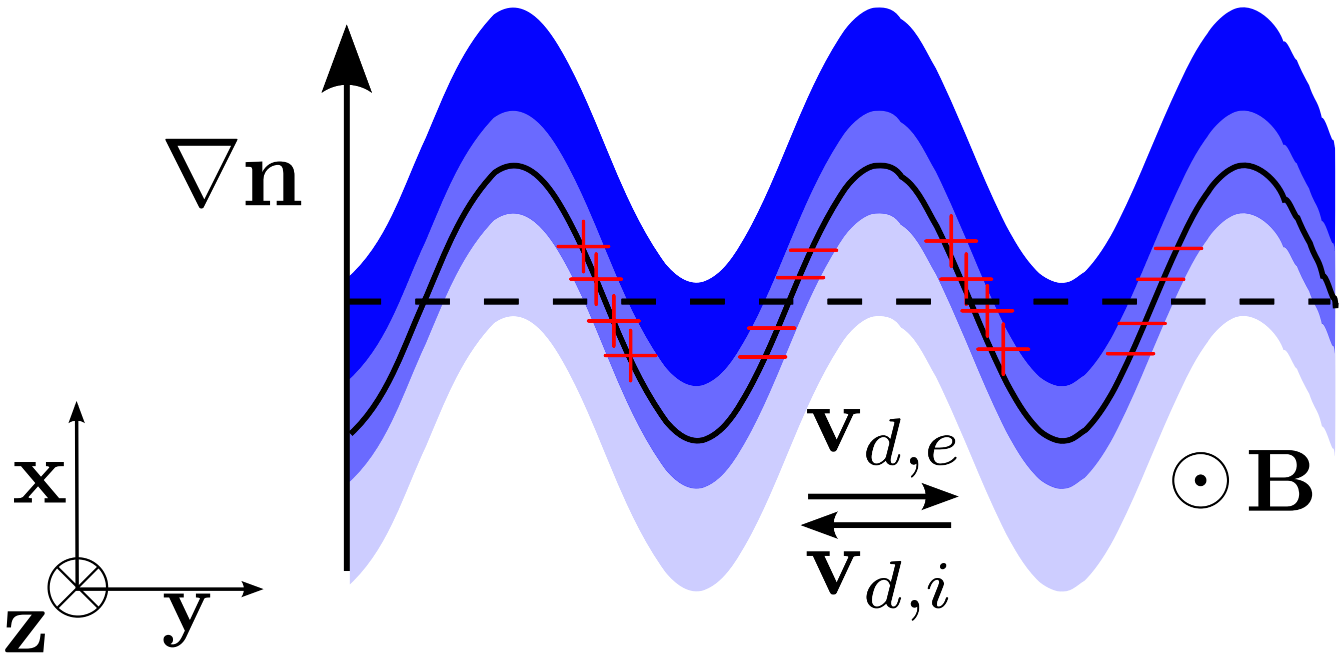
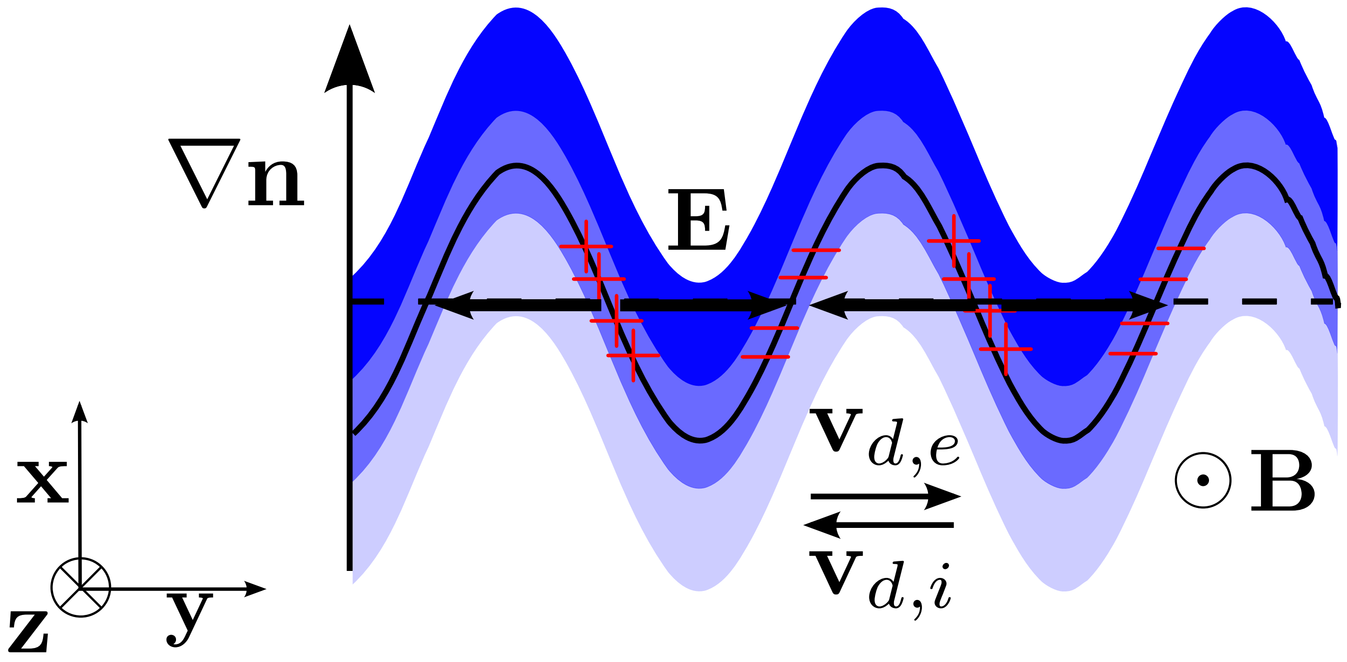


Drifts from curvature and gradient of the magnetic field, only non-vanishing for trapped particles

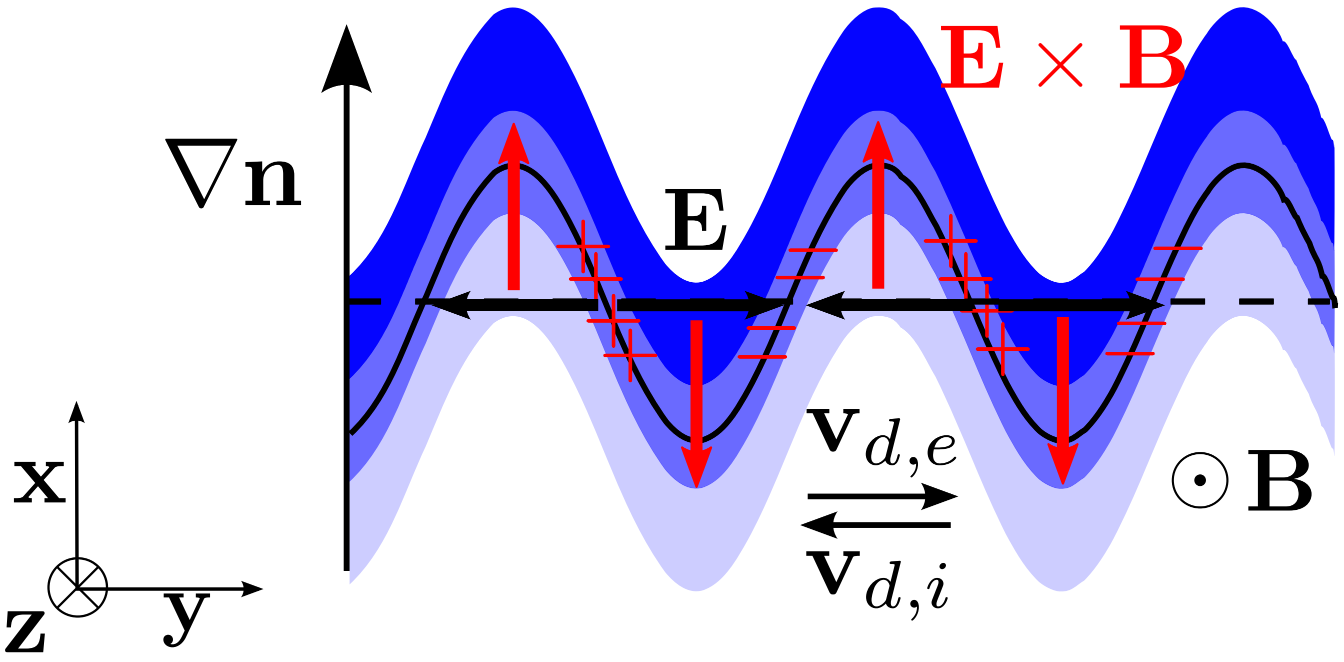
Trapped-particle mode



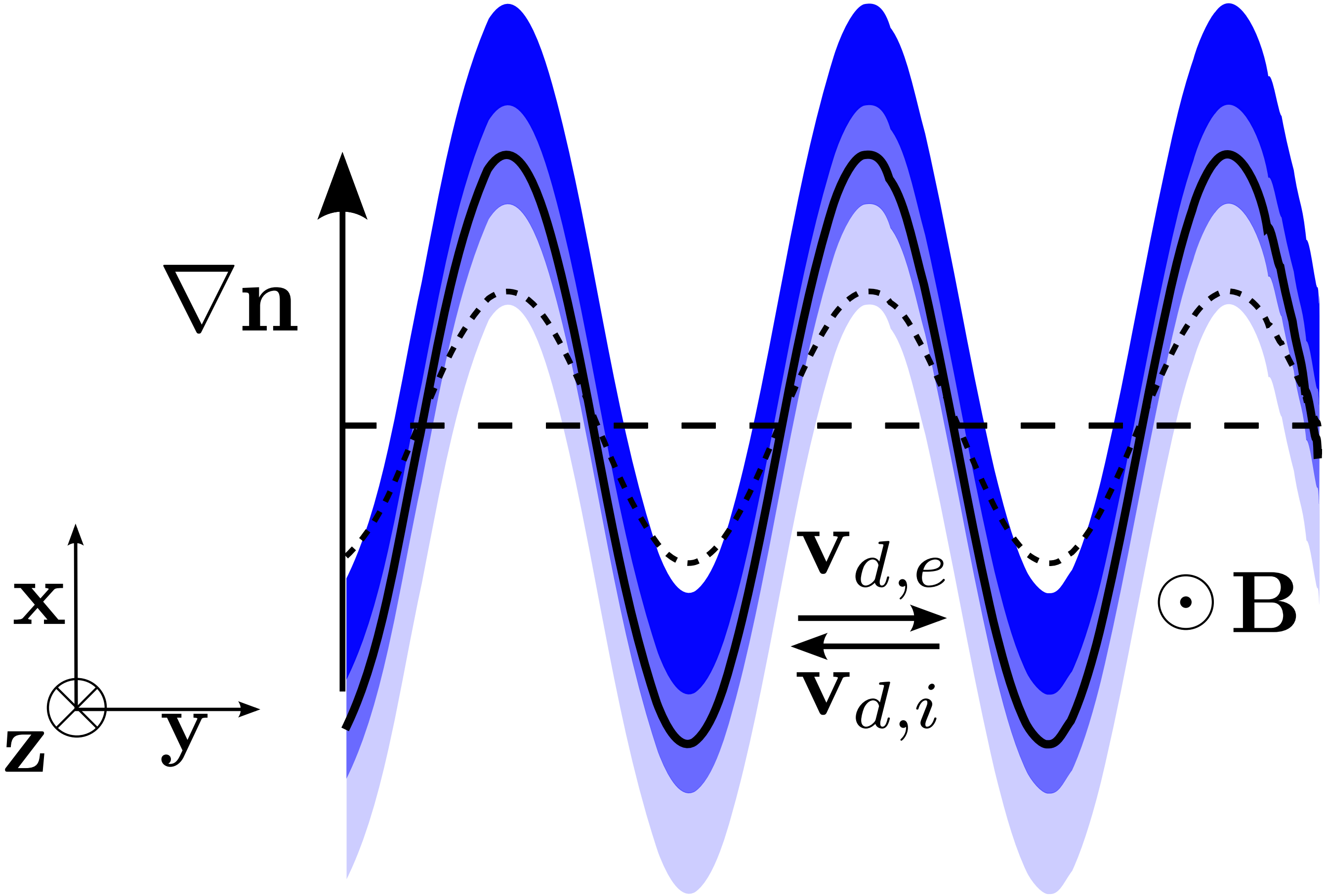
Trapped-particle mode



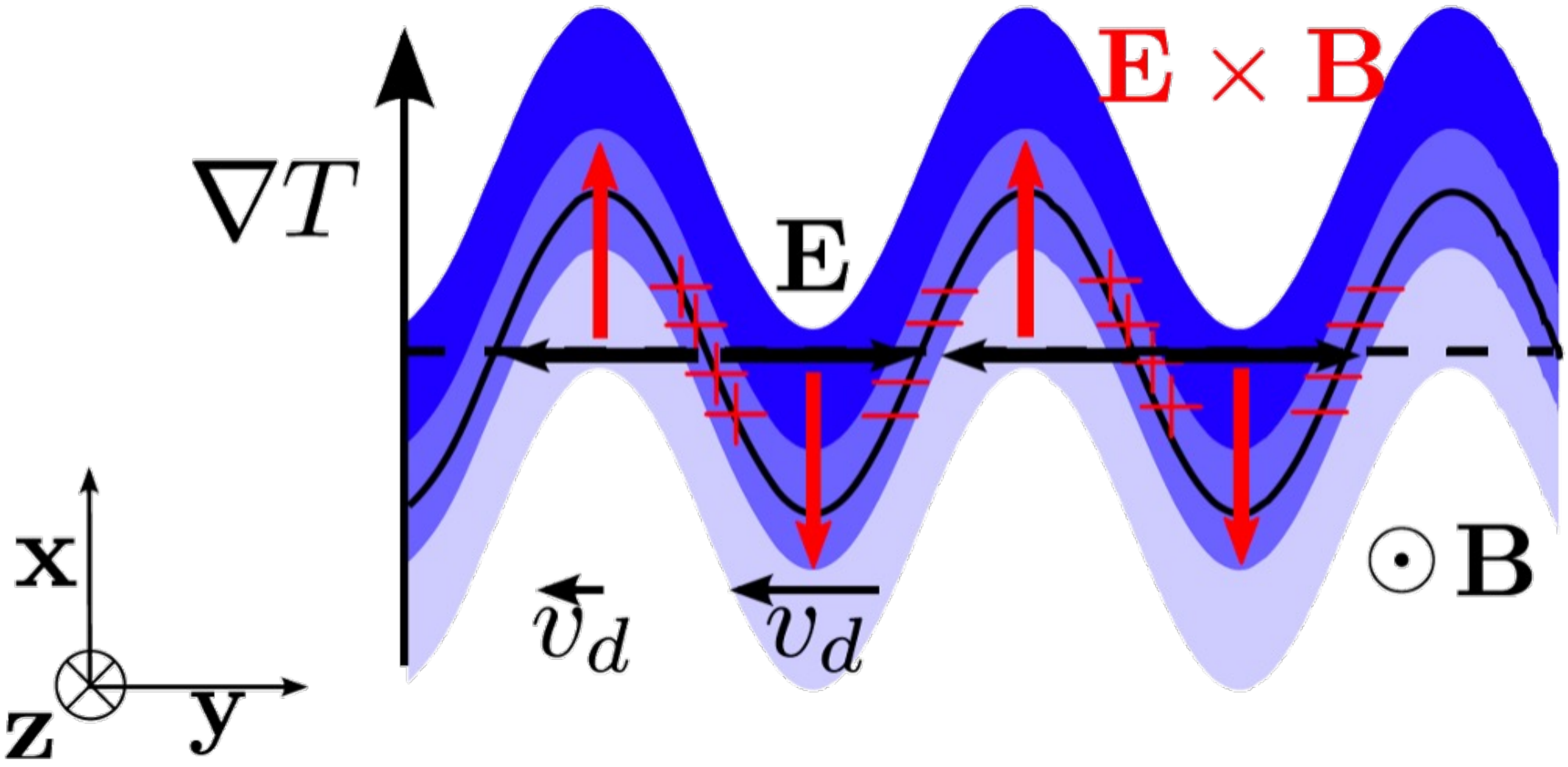
Trapped-particle mode



Trapped-particle mode



ITG yet again, but a different cartoon ☺



How would we actually calculate this?

Start from linearised, electrostatic gyrokinetic equation

$$v_{\|}\nabla_{\|}g_a - i(\omega - \omega_{da})g_a = -\frac{ie_a}{T_a}J_0\phi\left(\omega - \omega_{*a}^T\right)f_{a0}$$

iPad time ☺

The ITG dispersion relation

- Dispersion relation $D(\omega, \boldsymbol{k}_\perp, \varphi) \rightarrow D_0(\omega, \boldsymbol{k}_\perp, \varphi)$

$$\left(1 + \frac{T_i}{T_e}\right)\varphi(l) = \int \frac{\omega - \omega_{\star i}^T}{\omega - \omega_{di}} \frac{F_{Mi}}{n_o} J_0^2(k_\perp \rho_i)\varphi(l)\, \boldsymbol{d^3 v_i}$$

- Local approach

$$D_{loc} = \left(1 + \frac{T_i}{T_e}\right) - \int \frac{\omega - \omega_{\star i}^T}{\omega - \omega_{di}} \frac{F_{Mi}}{n_o} J_0^2(k_\perp \rho_i)\, \boldsymbol{d^3 v_i}$$

- Problematic → local growth rate & freq solution $\omega(l)$

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- Problematic → local growth rate & freq solution $\omega(l)$

After executing the integrals

Integral to solve: $$\frac{2}{\sqrt{\pi}}\int_{-\infty}^{\infty}\int_{0}^{\infty}\left(1+\frac{\omega_d-\omega_*^T}{\omega}-\frac{\omega_d\cdot\omega_*^T}{\omega^2}\right)\exp\left(-\left(x_\perp^2+x_\parallel^2\right)\right)x_\perp J_0^2 d\theta dx_\parallel dx_\perp$$

Have quadratic equation $$\omega=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

with
$$A=\Gamma_0(b)-\left(1+\frac{T_{0i}}{T_{0e}}\right)$$
$$B=\omega_*(1-\eta b)\Gamma_0(b)+\eta b\Gamma_1(b)+\frac{\hat{\omega}_d}{2}\left((2-b)\Gamma_0(b)+b\Gamma_1(b)\right)$$
$$C=\frac{\hat{\omega}_d\omega_*}{2}\left(\left(\frac{(2-b)\Gamma_0(b)+b\Gamma_1(b)}{2}\right)+\frac{\eta}{2}\left(\frac{2(b-1)^2\Gamma_0(b)+(3-2b)\Gamma_1(b)}{2}\right)\right)$$

Modified Bessel functions $\Gamma_n(b)=\exp(-b)I_n(b)$ $\quad b=\frac{(k_\perp\rho)^2}{2}$

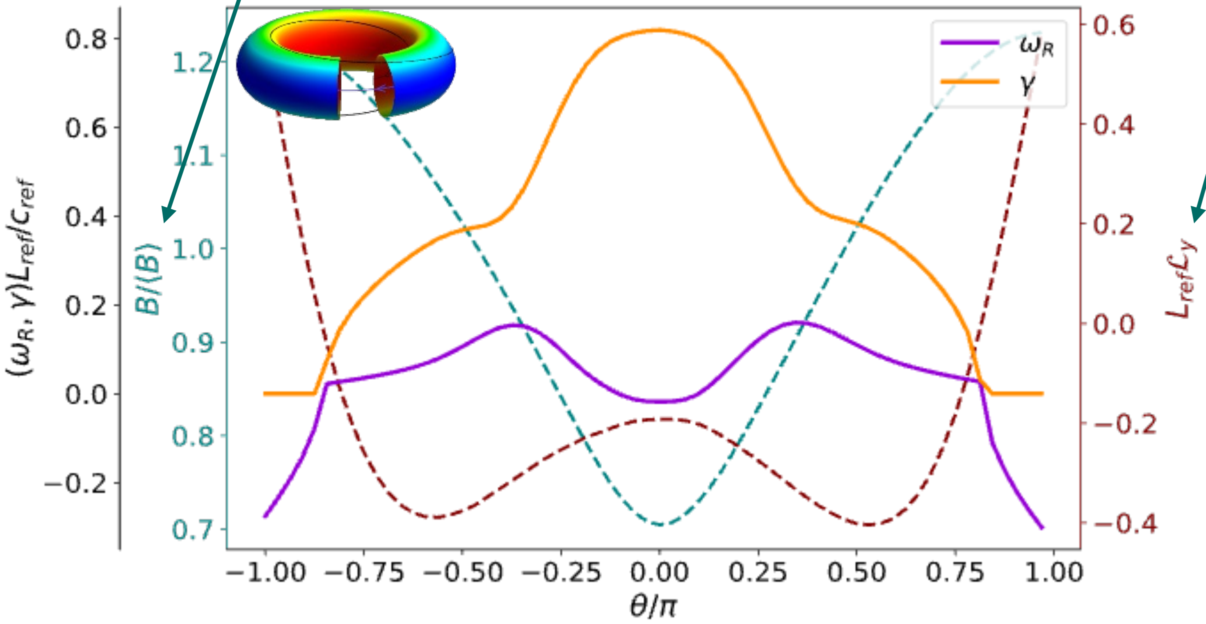
Obtaining a local solution for $\omega(k_\perp)$

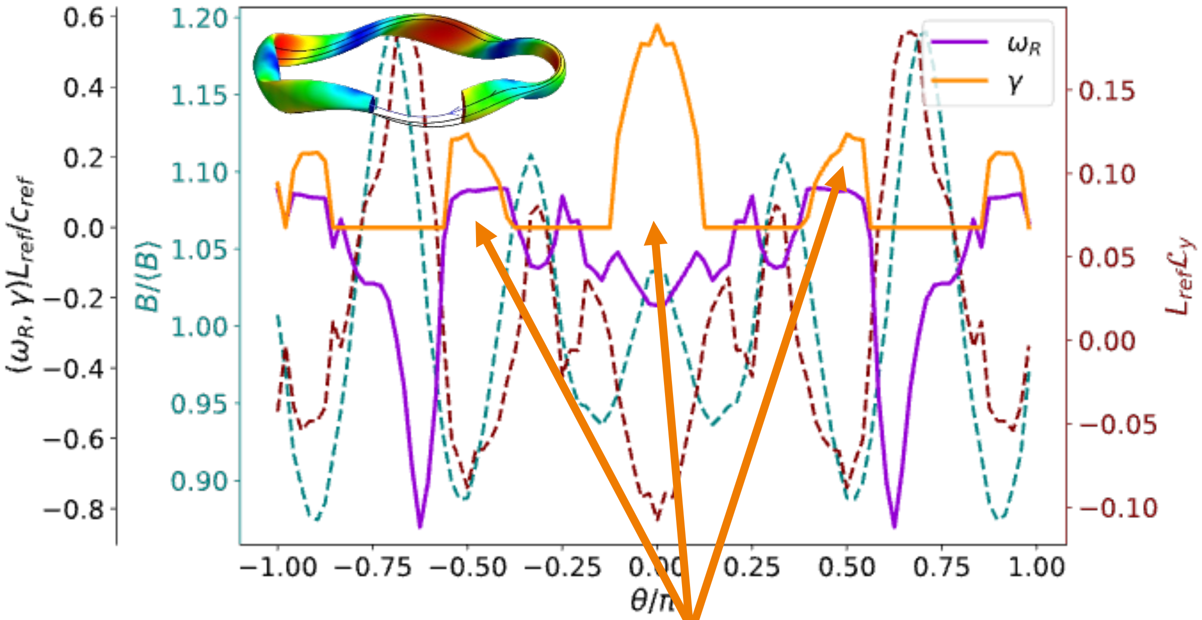
- Most obvious in strongly driven regime $\omega \sim \omega_{\star i} \gg \omega_{di}(l)$
- Expansion gives quadratic dispersion relation

$$A(l) + B(l)\omega + C(l)\omega^2 = 0$$

Magnetic field strength

"bad" curvature where negative





Large growth where curvature is bad!

ITG beyond the local approach - method

- Eigenmode physics → ω system property, not local quantity
- Neglected non-local quantity $D_{loc}(\omega, l)\varphi(l) = 0$
- Mode localisation should balance geometry variation

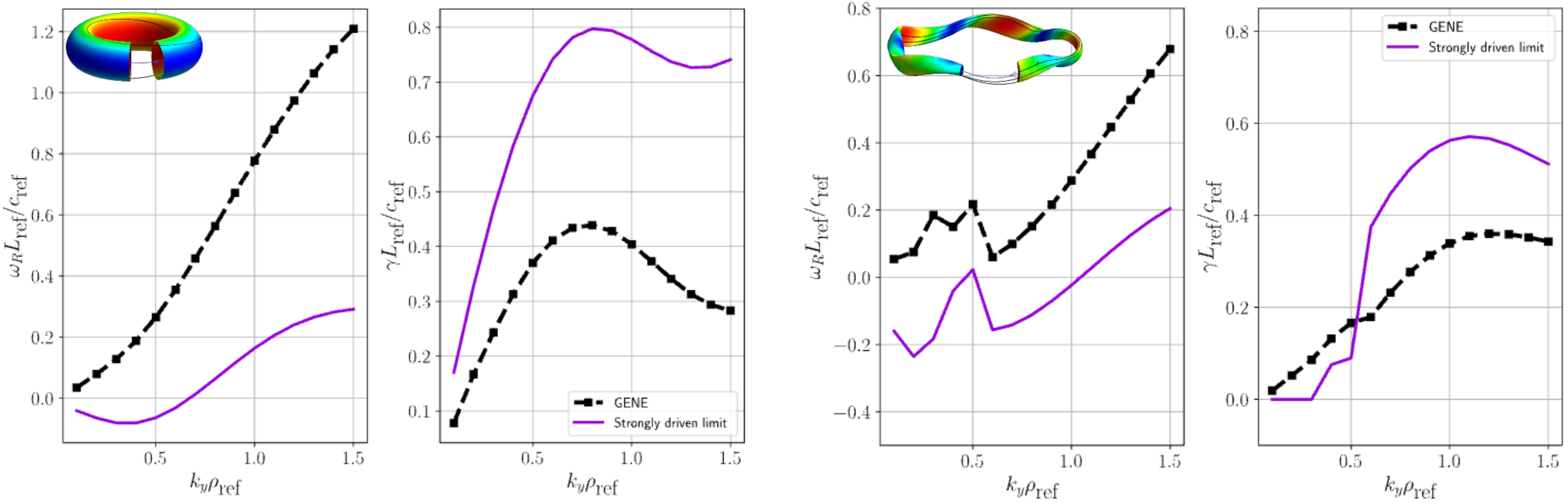
- Consider field-line global dispersion relation

$$D_{glob} = \int D_{loc}(\omega, l)|\varphi(l)|^2 \frac{dl}{B(l)}$$

- Zeros $D_{glob} = 0$ satisfy variational property [20]
- $\{\Re[D_{glob}], \Im[D_{glob}]\} = \{0,0\}$ → can solve for unknown $\Re\{\omega\}, \Im\{\omega\}$

ITG beyond the local approach – first results

- Parameters: $^{a}/_{L_n} = 1.5$, $^{a}/_{L_T} = 4.5$ and flux-tube at $s = 0.5$
- Using GENE $\varphi(\theta)$ → expect accurate reproduction of GENE ω (iff t-ITG)



ITG beyond the local approach – reintroducing the resonances

- Reintroduction of resonances in $D_{loc} \propto \int \frac{\omega-\omega_{\star i}^{T}}{\omega-\omega_{di}} \frac{F_{Mi}}{n_o} J_0^2(k_\perp \rho_i)\, \boldsymbol{d^3 v_i}$

$$\int \frac{\omega-\omega_{\star i}^{T}}{\omega-\omega_{di}} \frac{F_{Mi}}{n_o} J_0^2(k_\perp \rho_i)\, \boldsymbol{d^3 v_i} \rightarrow \Gamma_0(b) - \omega_{\star i}\left(1-\frac{3}{2}\eta_i\right)\mathcal{J}^0 + \frac{\omega_{\star i}}{2}\left(\frac{\omega_{\nabla B}}{\omega_{\star i}} - \eta_i\right)\mathcal{J}_\perp^2 + \omega_{\star i}\left(\frac{\omega_\kappa}{\omega_{\star i}} - \frac{1}{2}\right)\mathcal{J}_{||}^2$$

$$b = (k_\perp \rho_{Ti})^2$$

$$\omega_{di} = \omega_{\nabla B}\frac{v_\perp^2}{2v_{Ti}} + \omega_\kappa \frac{v_{||}^2}{v_{T_i}^2}$$

$$\omega = \omega_R + i\gamma$$

$$\omega_{\star i}^{T} = \omega_{\star i}\left(1+\eta_i\left(\frac{v^2}{2v_T^2} - \frac{3}{2}\right)\right)$$

$$\sigma_\gamma = \operatorname{sgn}\gamma$$

$$\mathcal{J}^0 = \frac{1}{i\sigma_\gamma}\int_0^\infty \frac{e^{i\sigma_\gamma \omega \xi}}{\sqrt{1+2i\sigma_\gamma \omega_\kappa \xi}} \frac{d\xi}{1+i\sigma_\gamma \omega_{\nabla B}\xi + b}$$

$$\mathcal{J}_\perp^2 = \frac{2}{i\sigma_\gamma}\int_0^\infty \frac{e^{i\sigma_\gamma \omega \xi}}{(1+2i\sigma_\gamma \omega_\kappa \xi)^{3/2}} \frac{d\xi}{1+i\sigma_\gamma \omega_{\nabla B}\xi + b}$$

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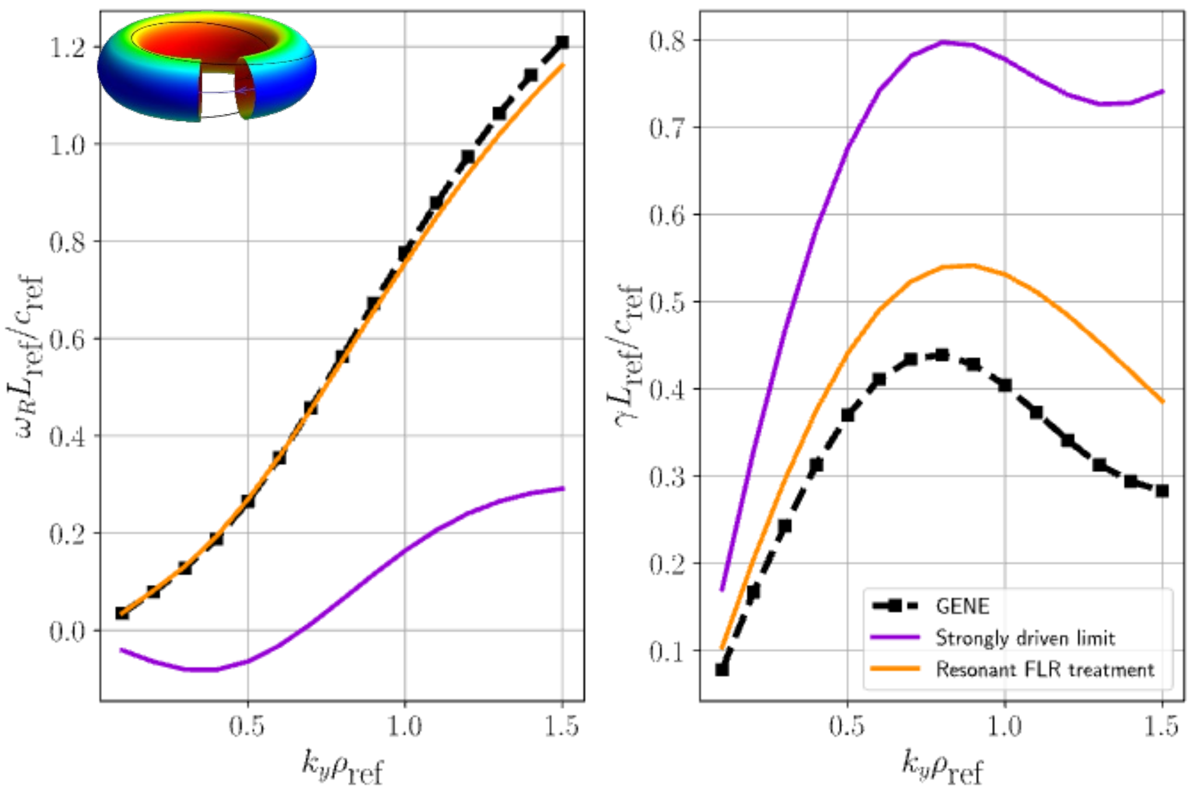
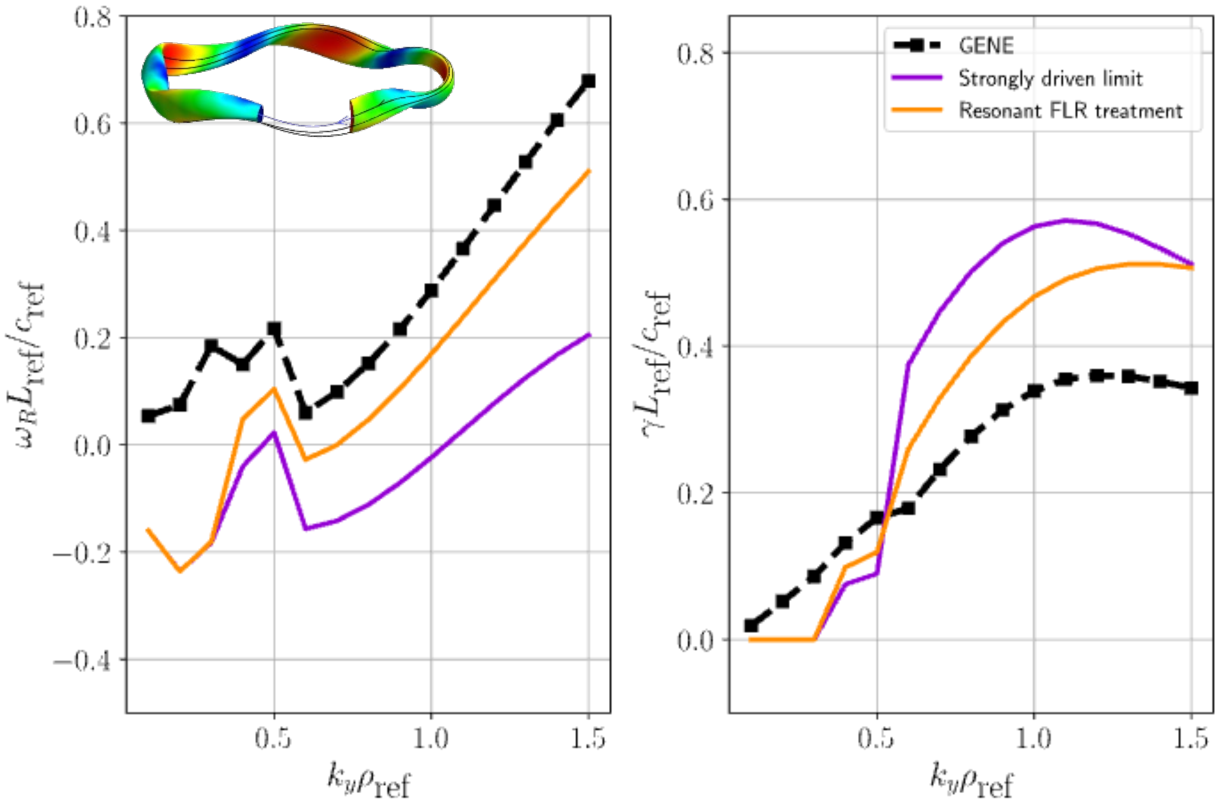
Beyond the global approach - results

- Including resonances → significant quantitative improvements





Technically: locality issues solved with keeping $v_\parallel \nabla_\parallel g_a$

$$v_\parallel \nabla_\parallel g_a - i(\omega - \omega_{da}) g_a = -\frac{ie_a}{T_a} J_0 \phi \left(\omega - \omega_{*a}^T\right) f_{a0}$$

Expand the parallel term (still ordered small, but not neglected)

$$g = J_0(z) f_M \frac{\omega - \omega_{*i}^T}{\omega - \bar{\omega}_{di}} \left[1 - i \frac{v_\parallel}{\omega - \bar{\omega}_{di}} \frac{\partial}{\partial l} - \frac{v_\parallel^2}{(\omega - \bar{\omega}_{di})^2} \frac{\partial^2}{\partial l^2} \right] \frac{e\tilde{\varphi}}{T_i}$$

[Wesson Tokamaks, Ch. 8 Eq. 8.3.5]

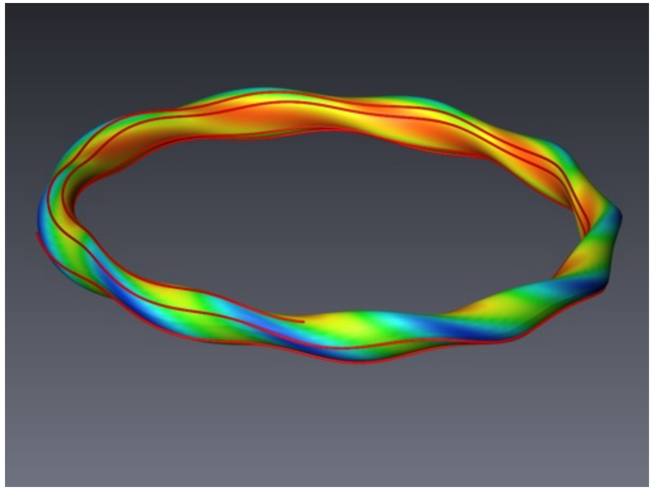
Now onto trapped-electron modes

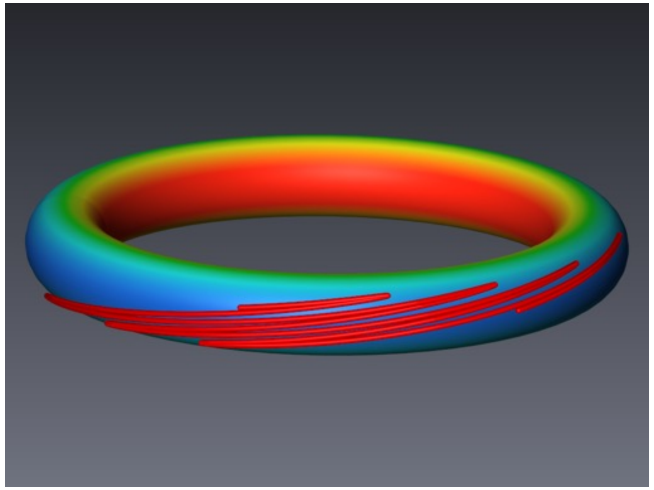
Start from linearised, electrostatic gyrokinetic equation

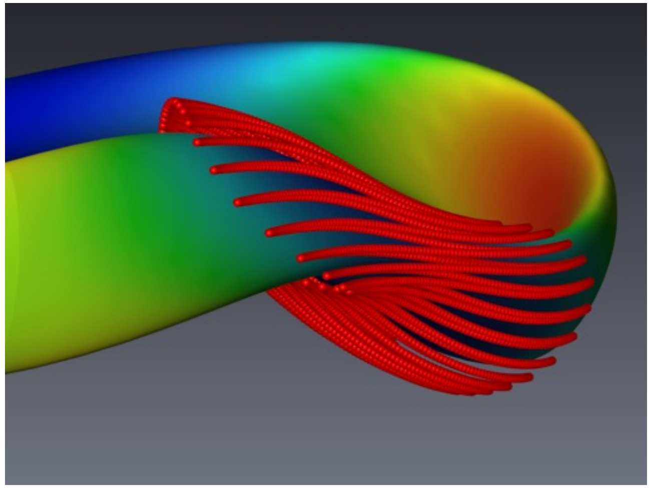
$$v_\parallel \nabla_\parallel g_a - i(\omega - \omega_{da}) g_a = -\frac{ie_a}{T_a} J_0 \phi \left(\omega - \omega_{*a}^T\right) f_{a0}$$

Passing particles don't really drift

Trapped particles do! Within the surface! → can resonate with drift waves







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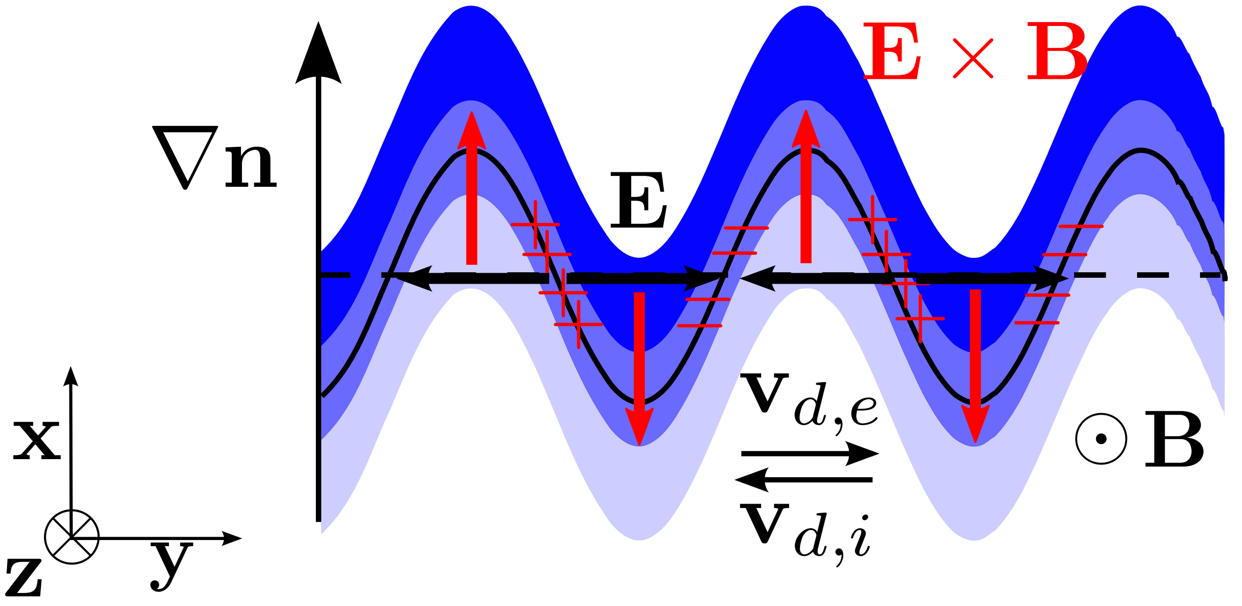


- Keep electron dynamics
- Only trapped electrons experience a drift on average
- Movement fast so can average over this bounce motion
- First, obtain the solution g_a

iPad time ☺

The trapped-particle mode (TPM)
(both ions and electrons are bouncing quickly compared with the mode frequency)

If the mode frequency is slow compared with the bounce motion of both ions and electrons,

$$\omega_{de,i} \ll \omega \ll \omega_{bi} \ll \omega_{be}$$

use the solution

$$g_a = \sqrt{2\epsilon}\frac{e_a}{T_a}\overline{J_0\phi}\frac{(\omega - \omega_{*a}^T)}{(\omega - \overline{\omega}_{da})}f_{a0}$$

And obtain the dispersion relation

$$\sum_a \frac{n_a e_a^2}{T_a}\phi = \sqrt{2\epsilon}\sum_a e_a \int_{\text{trapped}} \frac{e_a}{T_a}\overline{\phi}\frac{(\omega - \omega_{*a}^T)}{(\omega - \overline{\omega}_{da})}f_{a0}\mathrm{d}\mathbf{v}$$

The dispersion relation of the TPM

One time ϕ, one time with bounce average…

Multiply by $n_e e^2 \phi^* / Te$

And integrate along field line dl / B

Start from $$\sum_a \frac{n_a e_a^2}{T_a}\phi = \sqrt{2\epsilon}\sum_a e_a \int_{\text{trapped}} \frac{e_a}{T_a}\overline{\phi}\frac{(\omega - \omega_{*a}^T)}{(\omega - \overline{\omega}_{da})} f_{a0}\mathrm{d}\mathbf{v}$$

Assume small drift frequencies $$\frac{1}{\omega - \overline{\omega}_{da}} = \frac{1}{\omega}\left(1 - \frac{\overline{\omega}_{da}}{\omega}\right)^{-1} \simeq \frac{1}{\omega}\left(1 + \frac{\overline{\omega}_{da}}{\omega}\right)$$

And notice: $\omega_{*i} = -\omega_{*e}/\tau$ and $\omega_{di} = -\omega_{de}/\tau$ with $\tau = T_e/T_i$

Obtain $$(1+\tau)\frac{n_e e^2}{T_e}\phi = \frac{2n_e e^2}{\sqrt{\pi^3}}\int \overline{\phi} e^{-x^2}\left[\frac{1+\tau}{T_e}\left(1 - \frac{1}{\tau}\frac{\omega_{*e}^T \overline{\omega}_{de}}{\omega^2}\right)\right]\mathrm{d}\mathbf{x}$$ with $\mathbf{x} = \frac{v}{v_{Ta}}$

Now: convenient coordinates trick! New velocity coordinates!

$$\mathrm{d}\mathbf{v} = 2\pi v_\perp \mathrm{d}v_\perp \mathrm{d}v_\parallel = \sum_\sigma \frac{B\pi v^3 \mathrm{d}v\mathrm{d}\lambda}{|v_\parallel|} \qquad \lambda = \frac{v_\perp^2}{v^2 B} = \frac{\mu}{E} \qquad \sigma = \frac{v_\parallel}{|v_\parallel|}$$

Turning ϕ also into a bounce average

With defining different trapping wells n and unique bounce times $\tau_{bn}(\lambda) = \int_{l_1(n)}^{l_2(n)} \frac{\mathrm{d}l}{|v_\parallel|}$

$$\int_{-\infty}^{\infty} |\phi|^2 \frac{\mathrm{d}l}{B} = \frac{1}{\sqrt{\pi^3 v_{Te}^3}} \int_{-\infty}^{\infty} \overline{\phi}\phi^* \frac{\mathrm{d}l}{B} \sum_\sigma \int_0^\infty \pi v^3 \mathrm{d}v$$
$$\times \int_0^{1/B_{\min}} \mathrm{d}\lambda \frac{B}{|v_\parallel|} e^{-v^2/v_{Te}^2} \left[\left(1 - \frac{1}{\tau}\frac{\omega_{*e}^T \overline{\omega}_{de}}{\omega^2}\right)\right]$$
$$= \sum_\sigma \frac{1}{\sqrt{\pi^3 v_{Te}^3}}) \int_0^\infty \pi v^3 \mathrm{d}v \int_0^{1/B_{\min}} \mathrm{d}\lambda$$
$$\times \sum_n \int_{-l_1^{(n)}}^{l_2^{(n)}} \overline{\phi}\phi^* e^{-v^2/v_{Te}^2} \left[\left(1 - \frac{1}{\tau}\frac{\omega_{*e}^T \overline{\omega}_{de}}{\omega^2}\right)\right] \frac{\mathrm{d}l}{|v_\parallel|}$$

Obtain

$$\int_{-\infty}^{\infty} |\phi|^2 \frac{\mathrm{d}l}{B} = \frac{2}{\sqrt{\pi^3 v_{Te}^3}}) \int_0^\infty \pi v^3 \mathrm{d}v \int_0^{1/B_{\min}} \mathrm{d}\lambda \sum_n |\overline{\phi}|^2 \tau_{bn} e^{-v^2/v_{Te}^2} \left[\left(1 - \frac{1}{\tau}\frac{\omega_{*e}^T \overline{\omega}_{de}}{\omega^2}\right)\right]$$

Or

$$\omega^2 = -\frac{\frac{1}{\tau}\frac{1}{\pi^{3/2}}\frac{1}{v_{Te}^3} \int_0^\infty \pi v^3 \mathrm{d}v \int_0^{1/B_{\min}} \mathrm{d}\lambda \sum_n |\overline{\phi}|^2 e^{-v^2/v_{Te}^2} \omega_{*e}^T \overline{\omega_{de}} \tau_{bn}}{\int_{-\infty}^{\infty} |\phi|^2 \frac{\mathrm{d}l}{B} - \frac{1}{\pi^{3/2}}\frac{1}{v_{Te}^3} \int_0^\infty \pi v^3 \,\mathrm{d}v \int_0^{1/B_{\min}} \mathrm{d}\lambda \sum_n |\overline{\phi}|^2 e^{-v^2/v_{Te}^2} \tau_{bn}}$$

We can juggle with some inequalities…

$$\omega^2 = -\frac{\frac{1}{\tau}\frac{1}{\pi^{3/2}}\frac{1}{v_{Te}^3}\int_0^\infty \pi v^3 \mathrm{d}v \int_0^{1/B_{\min}} \mathrm{d}\lambda \sum_n |\overline{\phi}|^2 e^{-v^2/v_{Te}^2}\omega_{*e}^T\overline{\omega_{de}}\tau_{bn}}{\int_{-\infty}^{\infty} |\phi|^2 \frac{\mathrm{d}l}{B} - \frac{1}{\pi^{3/2}}\frac{1}{v_{Te}^3}\int_0^\infty \pi v^3 \,\mathrm{d}v \int_0^{1/B_{\min}} \mathrm{d}\lambda \sum_n |\overline{\phi}|^2 e^{-v^2/v_{Te}^2}\tau_{bn}}$$

If we define an inner product like this: $\langle g f\rangle = \int g^* f \frac{\mathrm{d}l}{|v_\parallel|}$

We can use Schwartz' inequality $|\langle g\, f\rangle|^2 \leq \langle f\, f\rangle\langle g\, g\rangle$ and thus $|\overline{\phi}|^2 \leq \overline{|\phi|^2}$

$$\begin{aligned}
&\frac{1}{\pi^{3/2}}\frac{1}{v_{Te}^3}\int_0^\infty \pi v^3 \,\mathrm{d}v \int_0^{1/B_{\min}} \mathrm{d}\lambda \sum_n |\overline{\phi}|^2 e^{-v^2/v_{Te}^2}\tau_{bn}\\
\leq&\frac{1}{\pi^{3/2}}\frac{1}{v_{Te}^3}\int_0^\infty \pi v^3 \,\mathrm{d}v \int_0^{1/B_{\min}} \mathrm{d}\lambda \sum_n \overline{|\phi|^2} e^{-v^2/v_{Te}^2}\tau_{bn}\\
=&\frac{1}{\pi^{3/2}}\frac{1}{v_{Te}^3}\int_0^\infty \pi v^3 \,\mathrm{d}v \int_0^{1/B_{\min}} \mathrm{d}\lambda \sum_\sigma \int_{-\infty}^{\infty} \frac{\mathrm{d}l}{|v_\parallel|}|\phi|^2 e^{-v^2/v_{Te}^2}\frac{B}{B}\\
=&\frac{1}{\pi^{3/2}}\frac{1}{v_{Te}^3}\int_{-\infty}^{\infty} \frac{\mathrm{d}l}{B}\sum_\sigma \int_0^\infty \pi v^3 \,\mathrm{d}v \int_0^{1/B_{\min}} \frac{\mathrm{d}\lambda B}{|v_\parallel|}|\phi|^2 e^{-v^2/v_{Te}^2}\\
=&\frac{1}{\pi^{3/2}}\frac{1}{v_{Te}^3}\int_{-\infty}^{\infty} \frac{\mathrm{d}l}{B}|\phi|^2 \int e^{-v^2/v_{Te}^2}\mathrm{d}^3 v\\
=&\int_{-\infty}^{\infty} |\phi|^2 \frac{\mathrm{d}l}{B}.
\end{aligned}$$

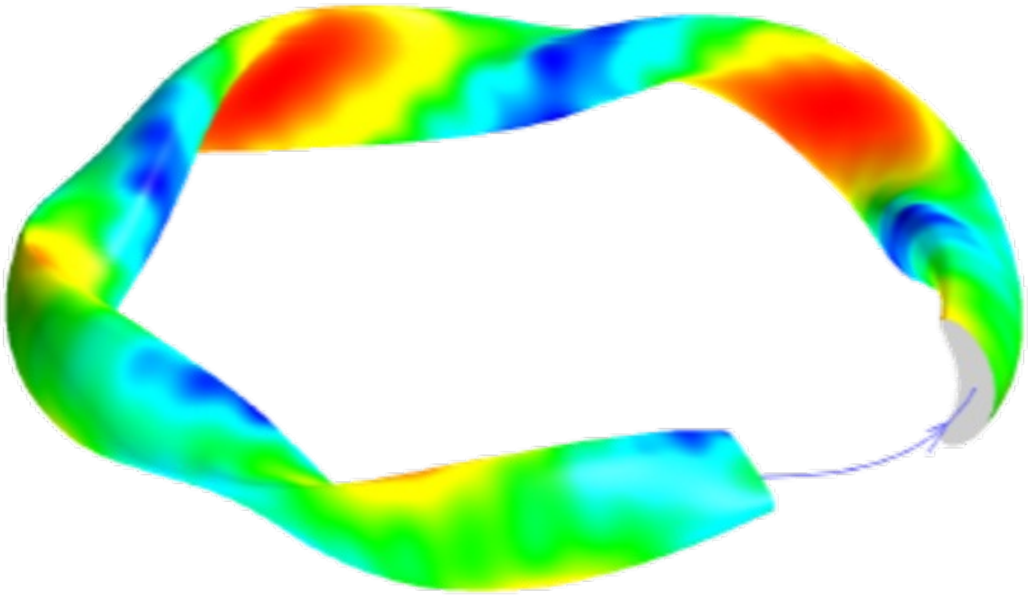
Denominator is always positive!

Sign of ω^2 depends on sign of numerator

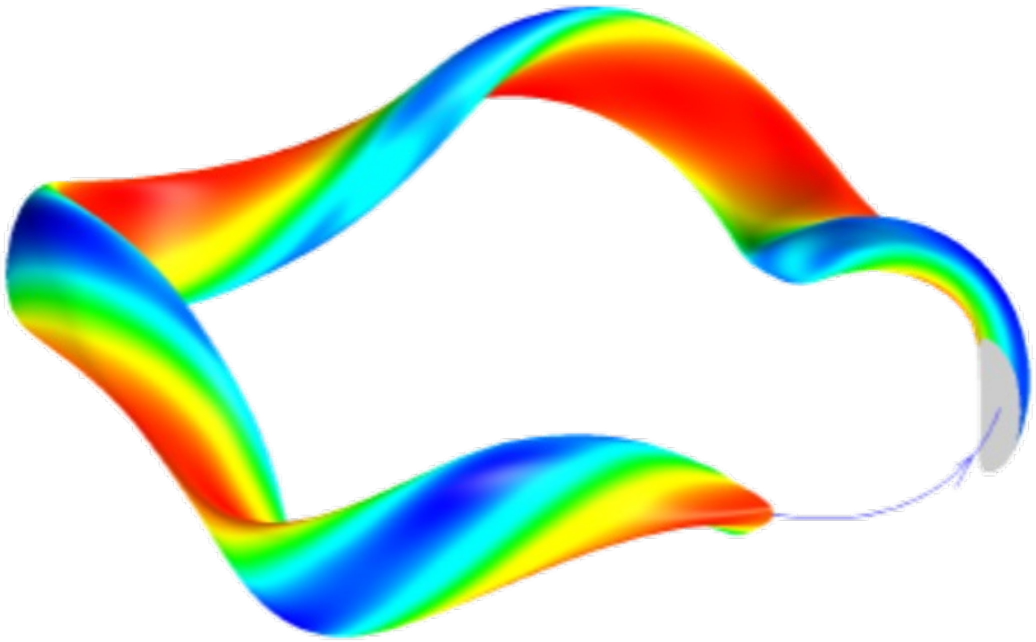
Which only depends on the sign of $\omega_{*e}^T\overline{\omega_{de}}$

TEMs in different stellarators

W7-X



HSX

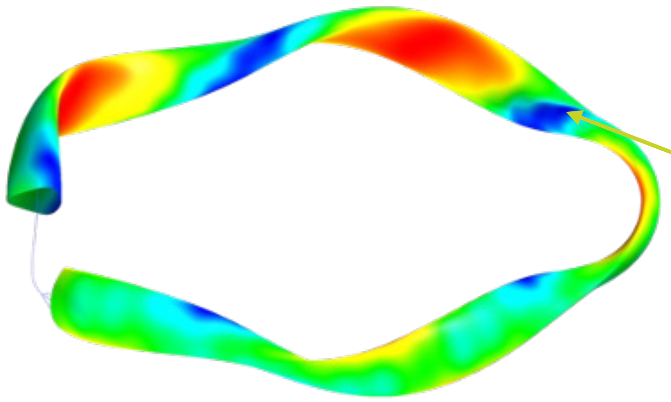


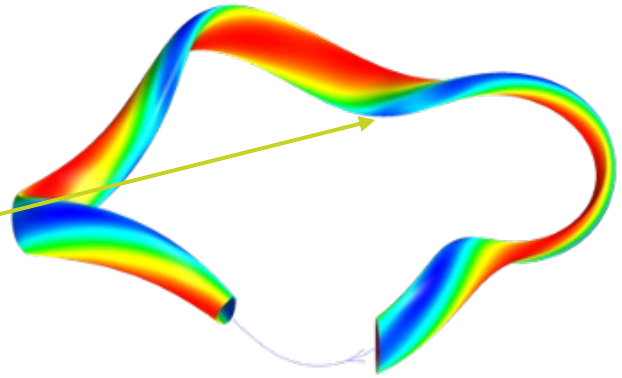
Trapped particles experience different curvature

Wendelstein 7-X

HSX

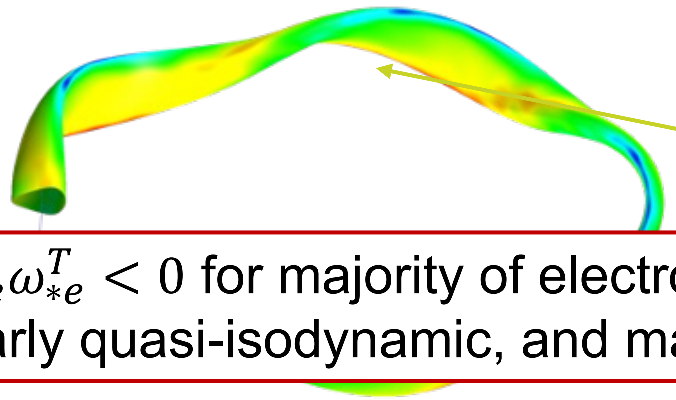
Magnetic field strength B

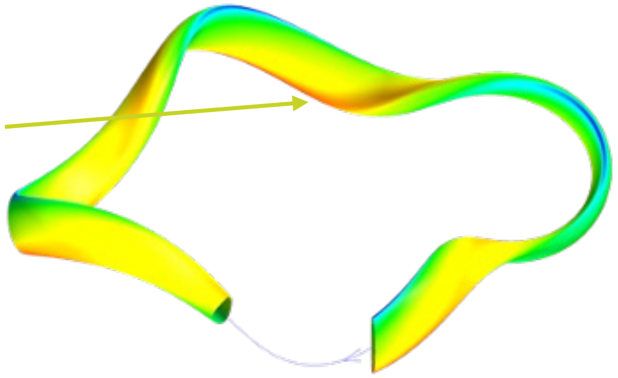




Location of trapped particles

Curvature κ

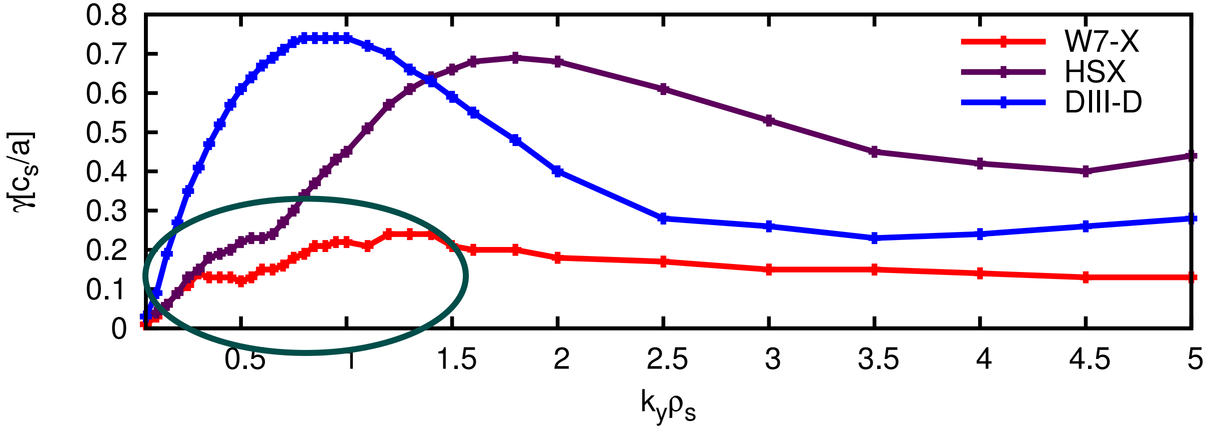




Location of strong curvature

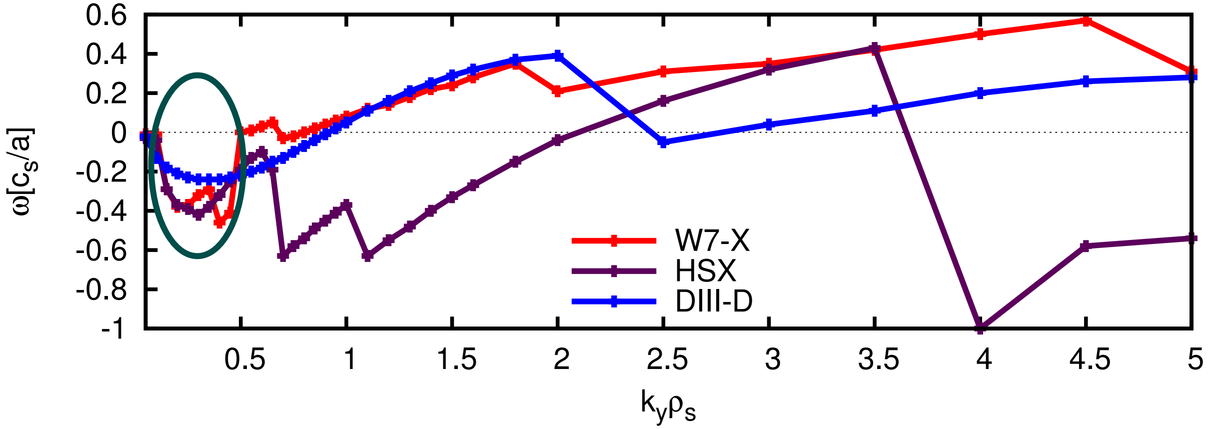
$\overline{\omega}_{de}\omega_{*e}^{T} < 0$ for majority of electrons
nearly quasi-isodynamic, and maximum-J

Trapped-electron modes (∇n −driven) - linear



Linear simulations with GENE with a/L_n=3

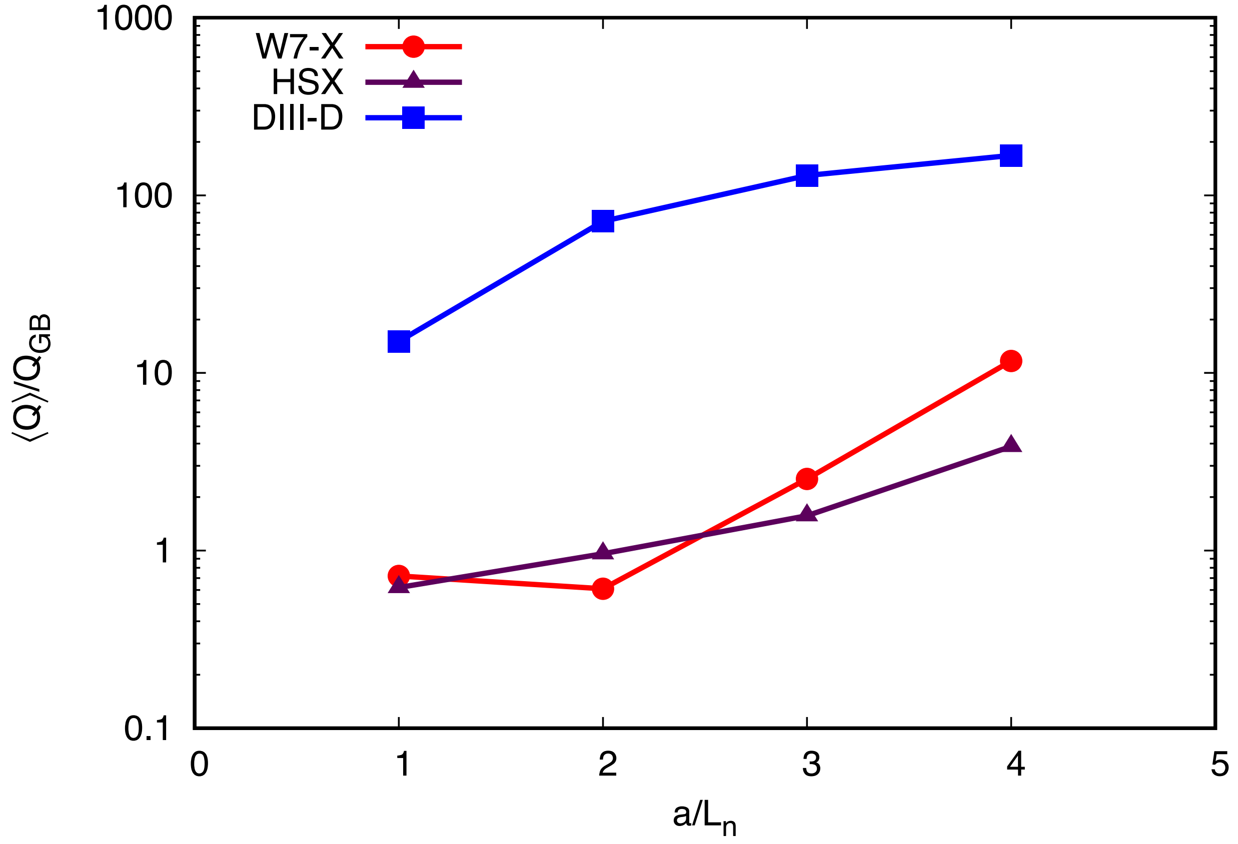
In W7-X:
generally small growth rates, mostly iTEM



In W7-X: typical TEM (or are they?)
(negative ω) only at smallest k

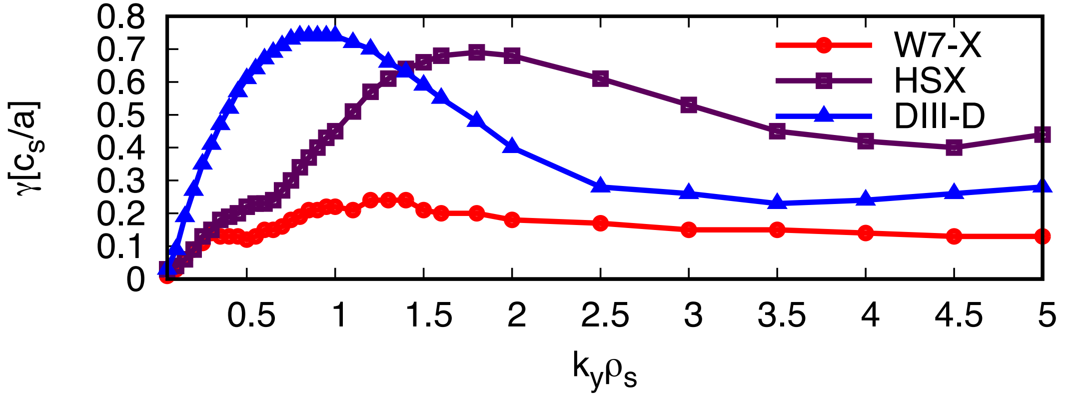
Electron-driven TEM indeed weak in W7-X

Trapped-electron modes (∇n –driven) - nonlinear



In nonlinear simulations:
W7-X has indeed low heat flux (even with surface-to-volume corrected)

Note: HSX heat flux also low
(c.f. high linear growth rates)



Powerful saturation mechanism?

More fancy ways to do the TPM calculation (and go to more arbitrary frequency orderings)

Not ignore the resonance, i.e. not order ω_d small

$$\frac{1}{\omega - \overline{\omega}_{da}} = \frac{1}{\omega}\left(1 - \frac{\overline{\omega}_{da}}{\omega}\right)^{-1} \simeq \frac{1}{\omega}\left(1 + \frac{\overline{\omega}_{da}}{\omega}\right)$$

- Obtain the same criterioin for TPMs to be stable: $\omega_{*a}\overline{\omega}_{da} < 0$
- But it's still only for $\omega_{de,i} \ll \omega \ll \omega_{bi} \ll \omega_{be}$

OR: go bold and super general with the solutions for g_a

$$\sum_\sigma g_{a,p}(l) = +\frac{e_a f_{a0}}{\pi T_a}\left(\omega - \omega_{*a}^T\right)\int_{-\infty}^{\infty}\frac{\mathrm{d}t}{\omega - t}\left[\int_{-\infty}^{\infty}\phi J_0 \cos M(t,l,l')\frac{\mathrm{d}l'}{|v_\parallel|}\right]$$

$$\sum_\sigma g_{a,t} = \frac{2e_a f_{a0}}{T_a}\frac{\omega - \omega_{*a}^T}{\sin\left(M(\omega, l_1, l_2)\right)}\int_{l_1}^{l_2}\frac{\mathrm{d}l'}{|v_\parallel|}\phi J_0 \times \cos\left(M(\omega, l_1, l_l)\right)\cos\left(M(\omega, l_u, l_2)\right)$$

And define an energy transfer rate $$\frac{\Delta E}{\Delta t} \propto -\int \mathrm{d}\lambda\,\omega_{*e}^T\overline{\omega}_{de}$$

Energy transfer from electrons to instability **taking ($\Delta E/\Delta t > 0$)** or **giving ($\Delta E/\Delta t < 0$)** ?

So if all electrons satisfy $\overline{\omega}_{de}\omega_{*e}^T < 0$
→ all electrons draw energy from the mode
→ no electron-driven TEMs can exist!

Conclusions and note of warning

- One can derive dispersion relations for ITGs and TEMs and learn quite a bit about stability properties and dependence on the geometry from those
- Common tricks are related to limiting yourself to distinct frequency ranges, which allows
  - kicking out terms of the GK equation because they're small or averaging them out
  - Ordering the drift frequency small
- Generally:
  - negative ("bad") curvature is … bad.
  - Overlap between bad curvature and magnetic minima is bad
  - Quasi-isodynamic configurations (W7-X approaches it) are great for TEMs
- **Can use these learnings for optimisation! A lot of work going on at the moment**
- **But: still a lot to do:**
  - **Zoo of other instabilities (Kinetic ballooning modes (EM), Microtearing modes (MTM), universal instabilities (UI)….**
  - **The linear results are only half the truth – need to understand saturation physics as well → nonlinear simulations are a safe bet CUE: Rogerio tomorrow ☺**