

# Numerical Modeling of Plasma Striations

2021 Princeton Plasma Physics Laboratory  
Graduate Summer School

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# What are plasma striations and why do we care?

- Formation of spatial patterns along the discharge current
  - light emission,  $n_e$ ,  $T_e$ , etc
- Multiple proposed mechanisms
  - e.g. non-linear dependence of ionization rate on electron density
- Observed experimentally for a long time but still not fully understood

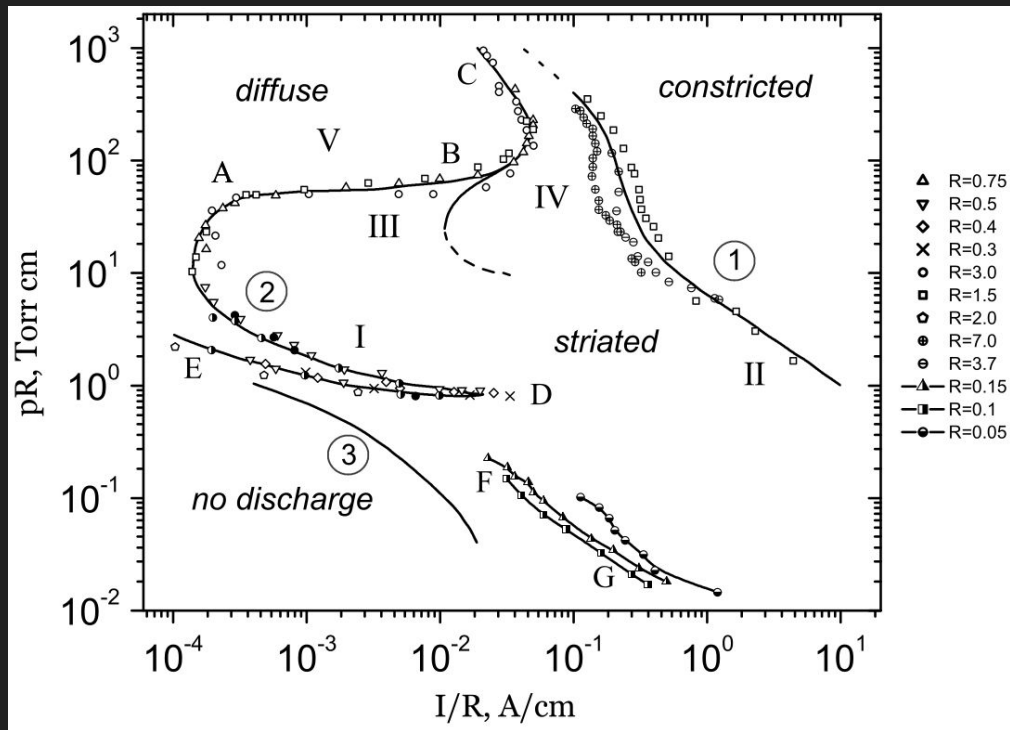


V. I. Kolobov et al, 2020 J. Phys. D: Appl. Phys. 53 25LT01 (5pp)



Courtesy of Dr Ed Thomas, Auburn University

# Current state of the field (1)

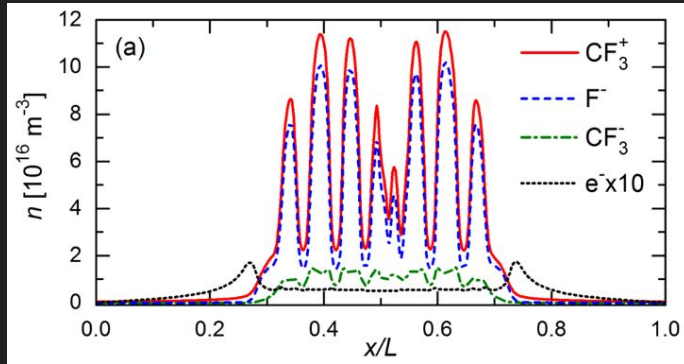


V. I. Kolobov, 2006 J. Phys. D: Appl. Phys. 39 R487-R506

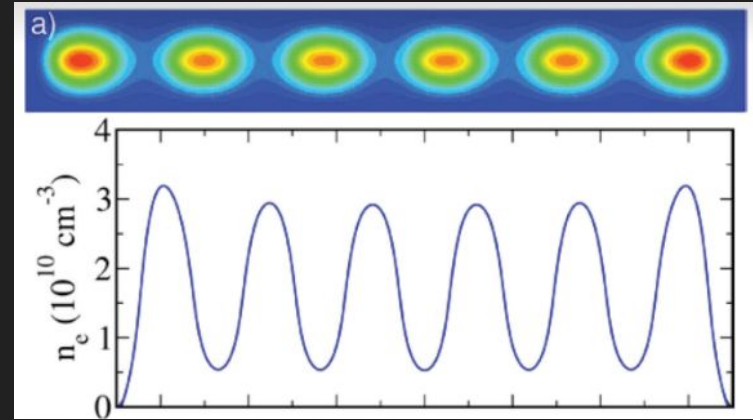
- Structures vary widely with experimental setup
  - discharge current ( $I$ )
  - gas pressure ( $p$ )
  - tube radius ( $R$ )
  - chemical composition
  - circuitry: DC, CCP, ICP, RF
  
- Scaling laws
  - plasmas with similar “combination” of parameters exhibit similar behavior

# Current state of the field (2)

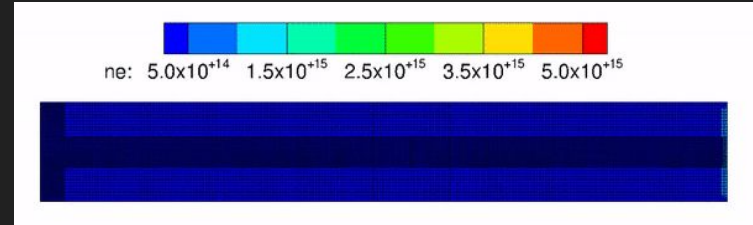
- Fluid simulations of striations along Pupp boundary obtained only recently
- PIC simulations have produced striations for lower current setups



Y.-X. Liu et al., 2016 Phys. Rev. Lett. 116 055024



V. I. Kolobov et al., 2020 J. Phys. D: Appl. Phys. 53 25LT01



R. R. Arslanbekov and V. I. Kolobov, 2021 Plasma Sources Sci. Technol. 30 045013

# Our research

## Non-local kinetic description for electrons

$$f(\mathbf{x}, \mathbf{v}, t) = f_0(\mathbf{x}, \mathbf{v}, t) + \mathbf{v} \cdot \mathbf{f}_1(\mathbf{x}, \mathbf{v}, t)/v,$$

$$v \frac{\partial f_0}{\partial t} + \nabla \cdot \Phi + \frac{\partial \Gamma_u}{\partial u} = v C_0^{inelastic},$$

$$\Phi = \frac{v^2 \mathbf{f}_1}{3} = -v D_r \left( \nabla f_0 - \frac{e \mathbf{E}}{w_0} \frac{\partial f_0}{\partial u} \right),$$

$$\Gamma_u = -\frac{e}{w_0} \mathbf{E} \cdot \Phi - (D_{elastic} + D_{ee}) \frac{\partial f_0}{\partial u} - (V_{elastic} + V_{ee}) f_0.$$

## Fluid description for ions

$$\frac{\partial n_+}{\partial t} + \nabla \cdot \Gamma_+ = S_+,$$

$$\Gamma_+ = -D_+ \nabla \cdot n_+ + \mu_+ \mathbf{E} n_+,$$

## Poisson eq. for electric field

$$\nabla \cdot (\nabla \varphi) = -\frac{e}{\epsilon_0} (n_+ - n_e),$$

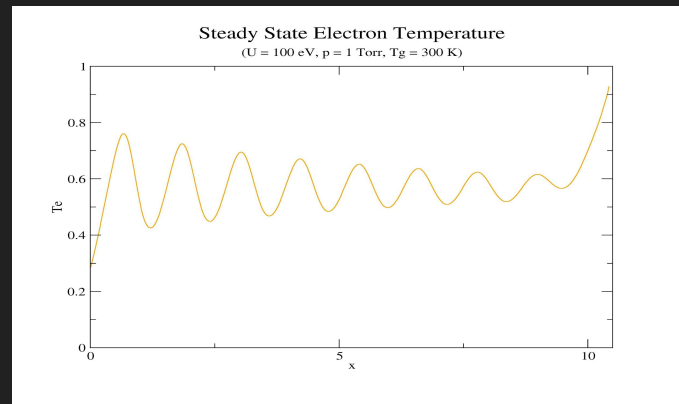
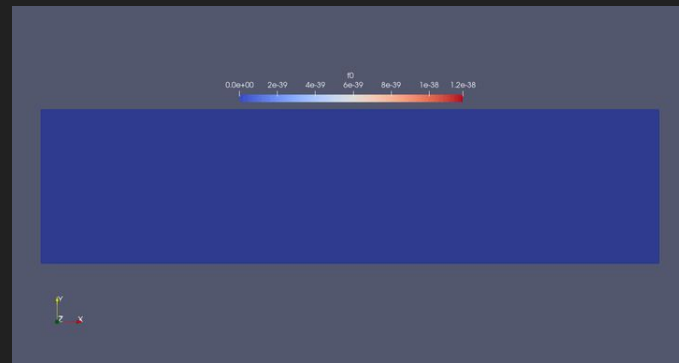
$$\mathbf{E} = -\nabla \varphi.$$

C. Yuan, et al., 1D kinetic simulations of a short glow discharge in helium, PHYSICS OF PLASMAS 24, 073507 (2017)

- Solving system while comparing capabilities of different softwares (COMSOL & Basilisk C)

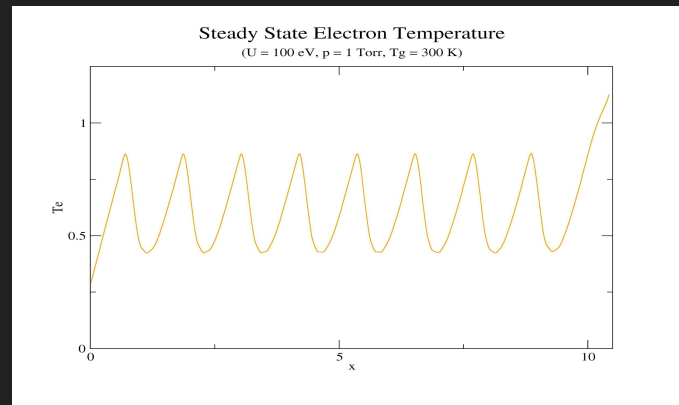
# Basilisk C

- Finite Volume method
- Explicit and implicit time treatment developed
- Use of limiters in explicit code in order to maintain  $f_0 > 0$
- Boundary conditions are non-trivial to implement due to full tensor diffusion

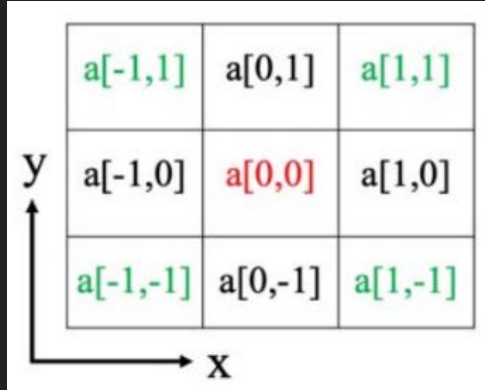


# Slope limiters

- Used to prevent unphysical negative values of  $f_0$  due to tangential fluxes
- Most limiters add numerical damping to the spatial profiles
- Superbee limiter (anti-diffusive) removes damping



# Time step coupling method (in Basilisk)



1. Start with  $f_0$ ,  $n_i$  and  $\varphi$  at time  $t$
2. Find  $f_0$  at time  $t + \Delta t$  using  $n_i$  and  $\varphi$  at time  $t$  (\*)
3. Find  $n_i$  at time  $t + \Delta t$  using  $f_0$  and  $\varphi$  at time  $t$  (\*\*)
4. Find  $\varphi$  at time  $t + \Delta t$  using updated  $f_0$  and  $n_i$  (\*\*)
5. Repeat from step (1.)

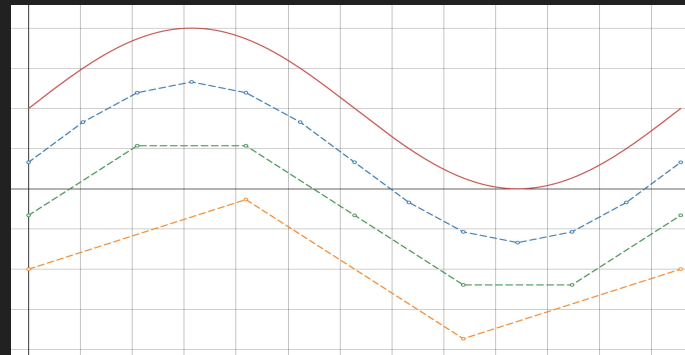
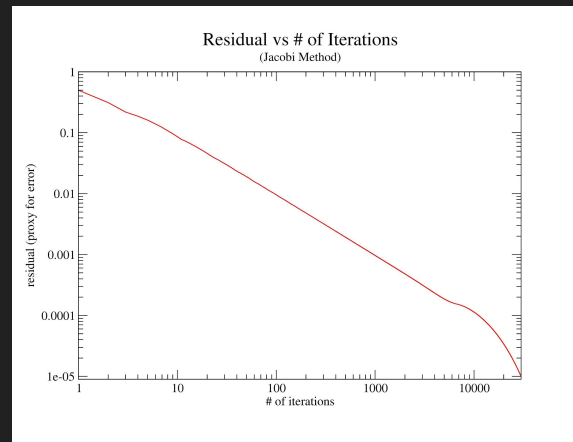
(\*) Diffusion-Reaction eq.  $\xrightarrow{\Delta t}$  Poisson-Helmholtz eq. (solved with multigrid method)

(\*\*) Tridiagonal system for 1D1E system



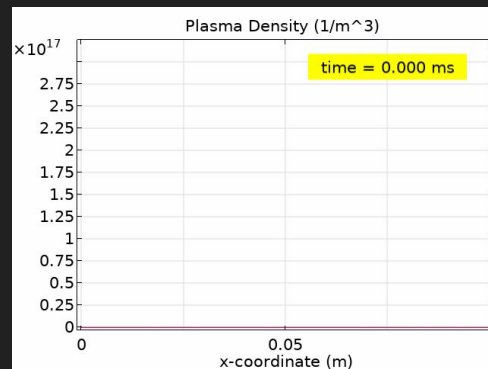
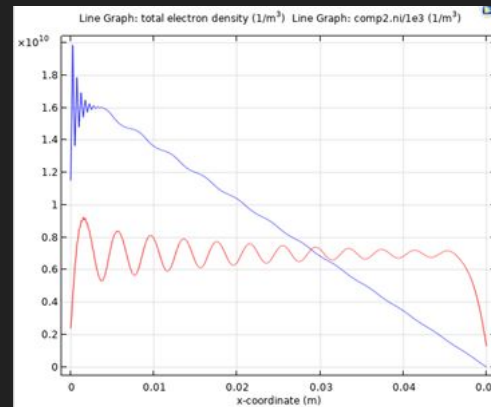
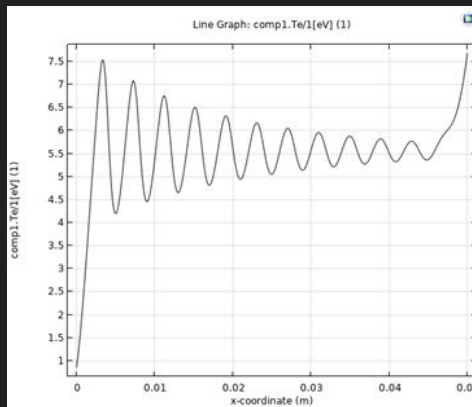
# Multigrid method

- Method of successive relaxations:
  - Discretized PDE = Linear system with one equation per cell/grid point
  - Solve recursively via Jacobi or Gauss-Seidel iterations
- Multigrid “enhancement”:
  - “Diminishing returns” in error reduction due to smooth errors persisting
  - Faster convergence by cycling through coarser and finer grids



# COMSOL

- Finite Element method
- Multiple options available for time stepping
- Natural log formalism in order to maintain  $f_0 > 0$
- Some features are “hidden” and not easily accessible, so it’s a bit of a “black box” at times



# Useful references

- Two-term spherical harmonic expansion: U. Kortshagen, C. Busch and L. D. Tsendin, On simplifying approaches to the solution of the Boltzmann equation in spatially inhomogeneous plasmas, PLASMA SOURCES SCIENCE AND TECHNOLOGY 5, 1 (1996)
- Multigrid method: W. L. Briggs, V. E. Henson, and S. F. McCormick A multigrid tutorial, 2nd edition (Jan 2000)
- Use of slope limiters in tensor diffusion: P. Sharma, G. W. Hammett, Preserving monotonicity in anisotropic diffusion, JOURNAL OF COMPUTATIONAL PHYSICS 227, 123-142 (2007)

# Thank you for your attention

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