Dynamo Basics

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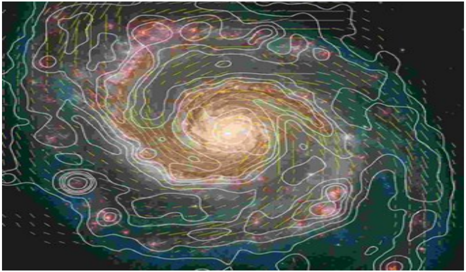
PPPL Graduate Summer School
Aug. 13, 2018

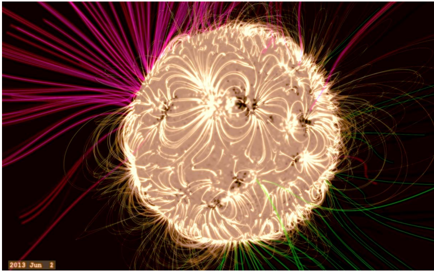
Magnetized universe: magnetic fields are observed to exist essentially everywhere in the universe

- Why is the universe magnetized?
- Observation of large-scale magnetic fields in widely different types of astrophysical objects

Galactic magnetic field



Solar/stellar magnetic field



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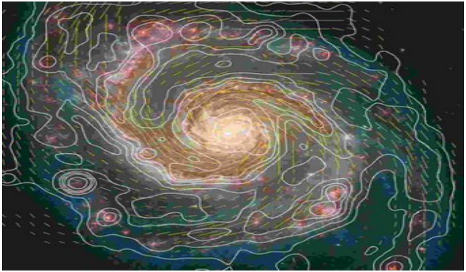

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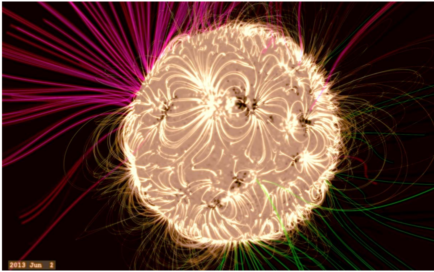

Sun is a natural dynamo engine making magnetic field and energy

(Loading sunexplosion.mp4)

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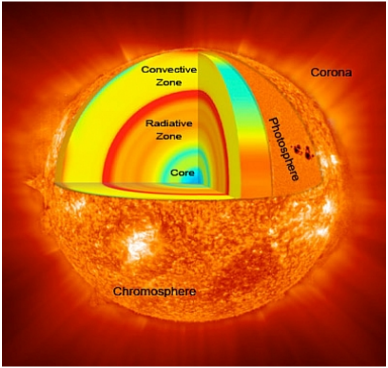
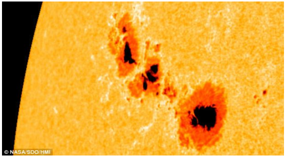
Different regions of sun

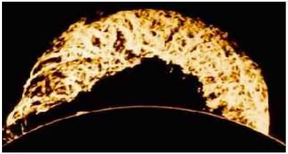


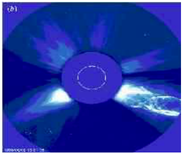
- Core: nuclear fusion extend from center to about 0.2 R (R=696,392 km) (T $\sim$ 15 million K)
- Radiative zone: heat transfer through thermal radiation (0.2R-0.7R) (T $\sim$ 2-7 million K)
- Convective zone: hear transfer through thermal convection from 0.7R to the surface (T $\sim$ 5800k- 2 million k)
- Photo-sphere: visible surface of sun with about 400km width (T $\sim$ 6000K)
- Solar atmosphere:
  1- chromosphere low temperatures (4000-100000 K), width about 2000km
  2- corona (halo during a total eclipse): high temperature about a few million K.
  3- Heliosphere regions beyond 20R.

Solar magnetic fields

Magnetic field has a key role in understanding major solar events.





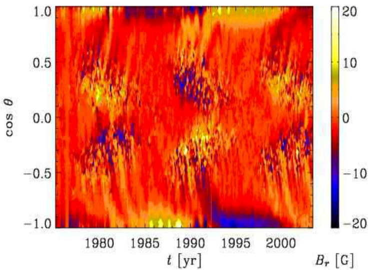


Sunspots: very intense magnetic lines of force break through the sun's surface (from 2011)

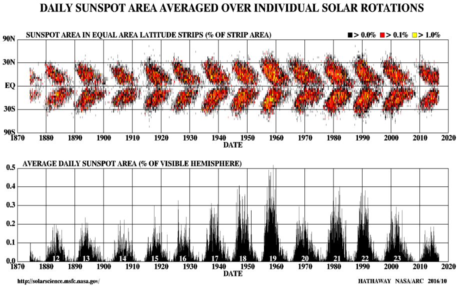
Prominences: long lasting magnetic structures above the surface of the sun (famous "Grand daddy" prominence of 4 June 1946)

Streamers and loops: structures in the corona shapes by the magnetic fields (big coronal mass eruption of 2 June 1998 from the LASCO chronograph)

Radial magnetic field alternates in time over the 11 year cycle and also changes sign across the equator.

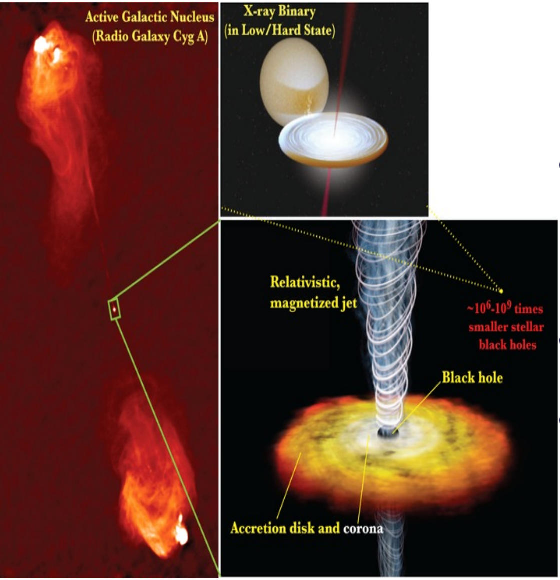


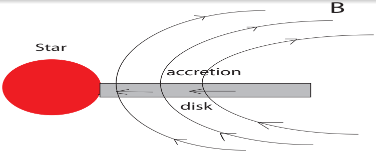
Longitudinally averaged radial component of the observed solar magnetic field as a function of cos(colatitude) and time. At the solar surface the azimuthally averaged radial field is only a few gauss.



Solar butterfly diagram showing the sunspot number in a space-time diagram. Peak magnetic field in sunspots of about 2 kG. Calculations predict field strengths of about 100kG in the deeper convection zone.

Magnetic field is required to cause accretion, the collimation of jets and star formation.



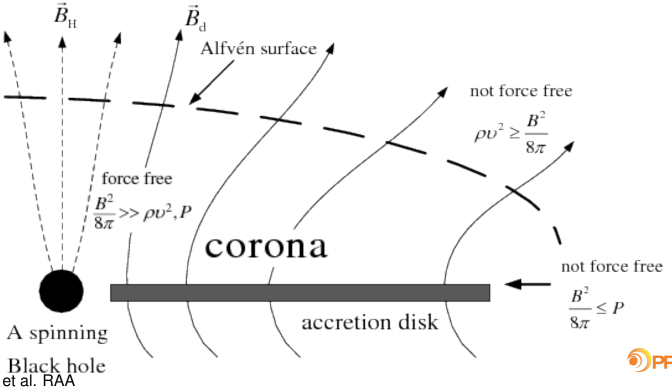


- What causes gas to be drawn in towards black holes, rather than remain in a stable orbit as planets do around the Sun? [S. Balbus]
- B is required for MHD instabilitie to cause accretion.
- Source of field: 1- May be supplied externally may be dragged from larger radii 2- maybe regenerated by local dynamo action

Self-organized plasmas: where dynamo and reconnection are linked

Examples of self-organized plasmas include
Flow-dominated: astrophysical disks (**dynamo**)
Magnetically-dominated: surface of stars (disk and stars coronas) (**reconnection**).

A disk-corona model



from Wang et al. RAA

Observed magnetic fields are believed to be caused by self-exciting dynamo

Dynamo: any device or media that can convert kinetic energy into electromagnetic energy

dynamo effect
kinetic energy $= = = = = = >$ magnetic energy

Self-exciting dynamo = No external fields or currents are needed to sustain the dynamo, aside from a weak seed magnetic field to get started.

Why should we care about dynamo problem?

- **Long standing physics challenge:**
  Explaining and understanding the origin of observed magnetic fields on all scales in the universe comprises a set of long standing questions in basic plasma astrophysics.

- **Important in laboratory fusion plasmas:**
  Plays important roles in laboratory plasmas including reversed field pinches, spheromaks and tokamaks.
  Local flux amplification/distribution due to the dynamo effect arising from correlated fluctuations can affect:
  - stability of the configuration
  - current transport
  - momentum transport

More on the lab applications in Tuesday's lecture (Prager) and some on Friday (F. E).

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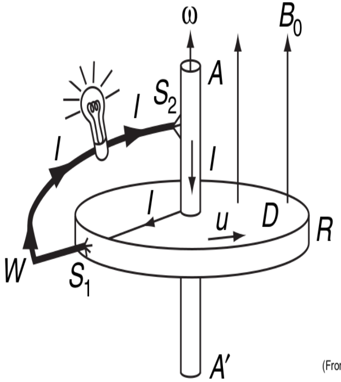
How do we make fields?

- Faraday (1831) discovered the principle of the electromagnetic generator - Faraday disk the first electric generator.



(From Roberts, 2007)

Electric conductor moving in a magnetic field

=> electromotive force => electric current => magnetic field

Current flows, and magnetic field is generated by current (I),

but not reinforcing B_0

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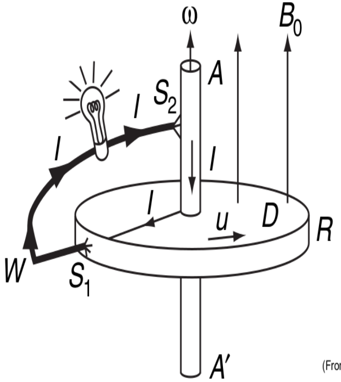

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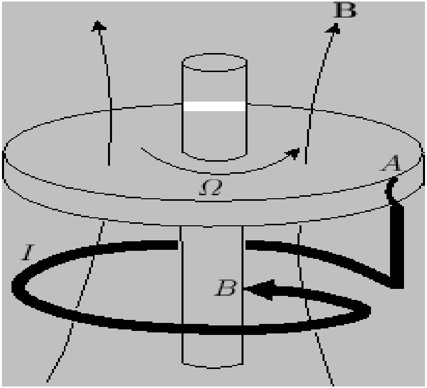
Current flows, and magnetic field is generated by current (I), **but not reinforcing** B_0

A simple disk dynamo model

- Larmor (1919) later proposed that the current generated by this process might be able to generate the magnetic field that the system needs. Could that explain solar fields?
  Self-excited dynamo



Possible when the wire loops around the disk (Bullard disk 1949)

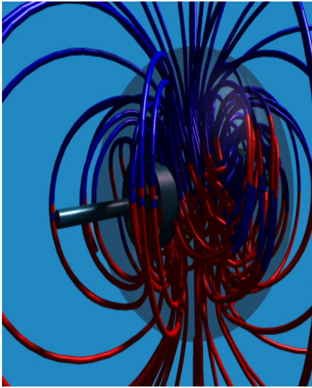
Magnetic field generated by current (I) can now **reinforce** B_0

Too simple to be applicable for astrophysical objects

In reality what do we need for a self-exciting dynamo?

To generate and sustain magnetic fields

- A weak **initial "seed" magnetic field** (which can be removed later).
- A suitable arrangement of the **motions** in an electrically conducting media and current pathways to produce magnetic field
- A **continuous supply of energy** to drive the electrical conductor sufficiently fast for self-excitation to be possible



Let's consider electrically conducting media/fluids, including plasmas. Start with Maxwell equations in the absence of magnetic diffusivities (the ideal case)

$$\text{Ampere's} \Rightarrow \mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

(Neglecting the displacement current $\mu\epsilon\partial_t E$)

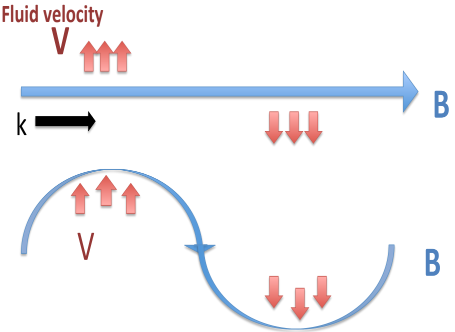
$$\text{Faraday's} \Rightarrow \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\text{Ohm's} \Rightarrow \mathbf{E} = -\mathbf{V} \times \mathbf{B}$$

Induction equation: (magnetic field responds to motion)

$$\frac{d\mathbf{B}}{dt} = \nabla \times (\mathbf{V} \times \mathbf{B})$$

Fluid elements move with the magnetic field.



$$\frac{d\mathbf{B}}{dt} = \nabla \times (\mathbf{V} \times \mathbf{B}) = -(\mathbf{V}.\nabla)\mathbf{B} + \underbrace{(\mathbf{B}.\nabla)\mathbf{V}}$$

bending of lines

Shear
Alfven waves: $\omega = \mathbf{k} \cdot \mathbf{V}_A$; $\mathbf{V}_A = \frac{\mathbf{B}}{\sqrt{\mu_0 \rho}}$

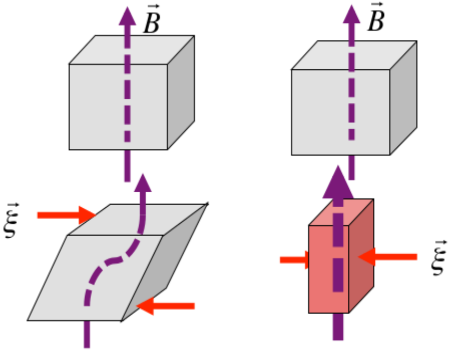
Hannes Alfven: 1970 Nobel Prize in physics for his pioneering work on MHD

Magnetic field can change by fluid motion

First an ideal case (diffusivities = 0)

$$\nabla \times (\mathbf{V} \times \mathbf{B}) = \underbrace{-(\mathbf{V}.\nabla)\mathbf{B}}_{\text{convection of fluid /advection}} + \underbrace{(\mathbf{B}.\nabla)\mathbf{V}}_{\text{stretching /bending of lines}} - \underbrace{\mathbf{B}(\nabla.\mathbf{V})}_{\text{compression}}$$

Compression increases B and perpendicular shear bends B
[figure from Sovinec MHD SULI talk]



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convection of fluid /advection

stretching /bending of lines

$\Downarrow$

compression

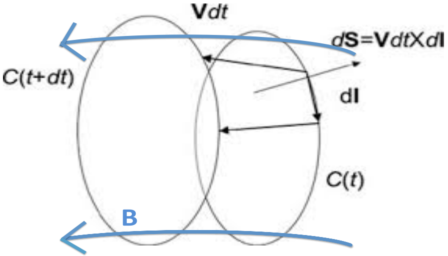
Example, consider a linear mean (large-scale) shear flow **V**= (0,Sx,0) and **B**=(B_0,0,0),

Solution from above $==>$ **B**= (1,St,0)B_0.

A field component in the direction of the flow grows linearly in time.

Stretching term due to flux freezing condition can lead to field amplification

Magnetic flux through a surface S, bounded by a curve C
$\Phi = \int \mathbf{B}.d\mathbf{S}$



Magnetic flux through a surface moving with the fluid remains constant in the high-conductivity limit.

Change in flux through a moving surface:
$\frac{d\Phi}{dt} = \int \frac{\partial \mathbf{B}}{\partial t}.dS + \int \mathbf{B}.\mathbf{V}xd\mathbf{l} = -\int [\nabla \times (\mathbf{E} + \mathbf{V} \times \mathbf{B}).d\mathbf{S} = 0$

(use $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$ and $\int (\mathbf{V} \times \mathbf{B}).d\mathbf{l} = \int \nabla \times (\mathbf{V} \times \mathbf{B}).d\mathbf{S}$)

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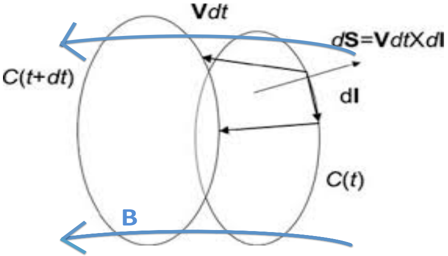


$$\frac{d\Phi}{dt} = 0$$

For a thin flux tube of length l and cross-section A,
any shearing motion which increases l will also amplify B;
An increase in l $==>$ decrease in A (because of incompressibility)
$==>$increase in B (due to flux freezing)

Let's put back magnetic diffusivity into the problem.

$$\text{Ampere's} \Rightarrow \mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

$$\text{Faraday's} \Rightarrow \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\text{Ohm's} \Rightarrow \mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{J}$$

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{V}.\nabla)\mathbf{B} = \; + \; (\mathbf{B}.\nabla)\mathbf{V} + \eta \nabla^2 \mathbf{B}$$

Magnetic field line stretching competes with diffusion

This effect can be measured with

magnetic Reynolds number $Rm = VL/\eta$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{V} \times \mathbf{B} + \frac{1}{Rm} \nabla^2 \mathbf{B} \right)$$

Magnetic energy evolution to measure the growth

Can measure growth in magnetic energy in the volume V

$$\frac{d\mathbf{B}}{dt} = \nabla \times [(\mathbf{V} \times \mathbf{B}) - \eta \mathbf{J}]$$

Dot product of B/μ_0 with the equation above and integration over the volume gives (HW):

$$\frac{d}{dt}\int \frac{B^2}{2\mu_0} dv = -\int \mathbf{V} \cdot (\mathbf{J} \times \mathbf{B}) dv - \int \eta J^2 dv - \oint \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} d\mathbf{s}$$

Change in magnetic energy **Ohmic dissipation**

It means: for the magnetic energy to grow, enough work must be done on the field by the fluid motion (against the Lorentz force) to overcome Ohmic dissipation.

similarly the change of kinetic energy can be shown (dot product of momentum equation with ρV and integration):

$$\frac{d}{dt}\int \rho \frac{V^2}{2} dv = \int \mathbf{V} \cdot (\mathbf{J} \times \mathbf{B}) dv + ...$$

The generation of magnetic energy goes at the expense of kinetic energy, without loss of net energy.

There is a critical Rm for different types of dynamos

$$\frac{d}{dt}\int \frac{B^2}{2\mu_0} dv = -\int \mathbf{V}\cdot(\mathbf{J}\times\mathbf{B})dv - \int \eta J^2 dv$$

For a laminar flow (not small-scale), we can have estimates of their magnitudes,

$\int \mathbf{V}\cdot(\mathbf{J}\times\mathbf{B})dv \approx (VJB)L^3 \sim V(B/\mu_0 L)BL^3 = V/L(B^2L^3/\mu_0)$
$\approx V/L(W_B)$

and
$-\int \eta J^2 dv \sim \eta J^2 L^3 \sim (\eta/\mu_0)(B^2L^3/\mu_0)/L^2 \approx (\eta/\mu_0)(1/L^2)(W_B)$

For $\frac{d}{dt}\int \frac{B^2}{2\mu_0}dv > 0 ==> Rm \equiv VL/\mu_0\eta > 1$

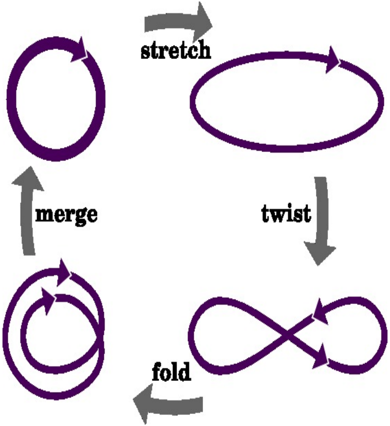
$\hookrightarrow$ **For stretching to win over the diffusion (reconnection)**

$==>$ $\boxed{\mathrm{Rm} > \mathrm{Rm}_{\mathrm{crit}}}$

Let's put all these pieces of physics together to get magnetic self-enforcement

Zeldovich "Stretch-twist-fold" dynamo : Fast dynamos

The model (1975) illustrates several features of more realistic dynamos.



Stretch: Cross section decreases by 2 (flux freezing) $==>$ B doubles

Twist: rope is twisted into figure 8

Fold: then fold, now there are two loops, field points in the same direction (similar volume). Flux through this volume has now doubled!

Two loops merge into one with the help of small diffusivity (process irreversible!)

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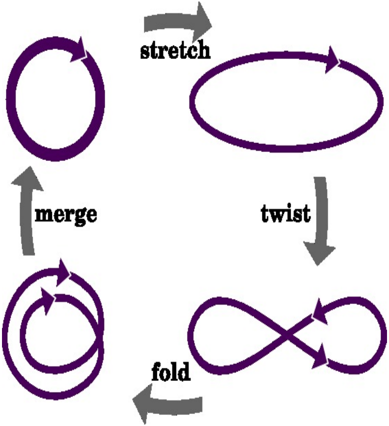

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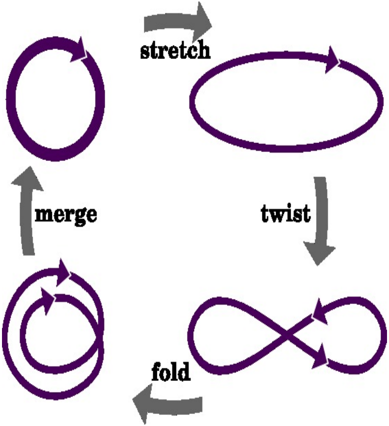

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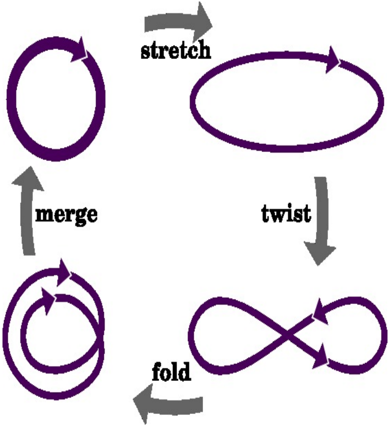


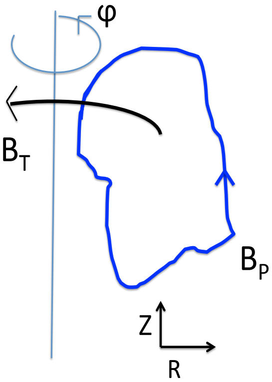
Stretch:To amplify field, **sheared motion** needed

Twist: Twist is needed for the mean field to leave the plane (go to **third direction**)

If only stretch and fold fields cancel.

Need for third dimension: the basis of Cowling theorem

Cowling's theorem: dynamo action in 2-D is not possible



Consider an axisymmetric system ($\partial/\partial\phi = 0$)

$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} = (\mathbf{V} \times \mathbf{B}) - \eta \mathbf{J} \qquad (1$$

Let $\psi = RA_\phi$ $\quad(\mathbf{B} = \frac{\nabla\psi}{R} \times \hat{\phi})$
or $RB_R = -\frac{\partial\psi}{\partial z}$, $RB_z = \frac{\partial\psi}{\partial R}$
and $J_\phi = -\nabla \cdot (\frac{1}{R}\nabla\psi)$
The toroidal component of Eq. 1 becomes:

$$\frac{\partial\psi}{\partial t} = \mathbf{V} \times [\frac{\nabla\psi}{R} \times \hat{\phi}]_\phi + \eta\nabla \cdot (\frac{1}{R}\nabla\psi)$$

$$\frac{\partial\psi}{\partial t} = -V_p \cdot \nabla\psi + \eta\nabla \cdot (\frac{1}{R}\nabla\psi) \qquad (2)$$

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Cowling's theorem: dynamo action in 2-D is not possible

Multiply Eq. 4 by ψ and integrate over space:

$$\frac{\partial}{\partial t}\int \psi^2 dV = -\int \underbrace{\psi V_p \cdot \nabla\psi}_{\hookrightarrow \nabla\cdot(V_p\psi^2/2)} dV + \eta\int \psi\nabla\cdot(\frac{1}{R}\nabla\psi)dV \qquad (3)$$

RHS becomes:

$$= -\int \frac{\psi^2}{2} V_p \cdot \hat{\mathbf{n}} dS - \eta\int \frac{1}{R}\psi\hat{\mathbf{n}}\cdot\nabla\psi dS - \eta\int \frac{1}{R}|\nabla\psi|^2 dV$$

The surface integrals vanish if $V_p \cdot \hat{\mathbf{n}} = 0$ or when scales as $1/r^2$ as $r- > \infty$ we left with:

$$\frac{\partial}{\partial t}\int \psi^2 dV = -\eta\int \frac{1}{R}|\nabla\psi|^2 dV$$

Cowling's theorem: dynamo action in 2-D is not possible

$$\frac{\partial}{\partial t} \int \psi^2 dV = -\eta \int \frac{1}{R} |\nabla \psi|^2 dV$$

The left hand side must continually decrease with time (as ψ^2 and B_p must vanish as $t \to \infty$), as the right hand side is positive definite (flux decays and impossible in axisymmetric systems).

Other anti-dynamo theorems: A "planar" flow, of the form (u(x,y,z,t),v(x,y,z,t),0), cannot maintain a magnetic field (Zeldovich's theorem), also dynamo is impossible from purely toroidal flows.

Dynamo requires some ingredient that is symmetry breaking.

1- At what scales magnetic field/energy are generated?
What is the peak of magnetic energy spectrum?

- "Small-scale dynamo" – sustainment of magnetic energy due to magnetic fluctuations in a turbulent state (with a small amount of flux).
  Example: galactic clusters
  SSD : produces scales below or of the order of forcing scale
- "Large-scale dynamo"– the generation of large-scale field accompanied by the generation of total magnetic flux.
  Examples: Systems with rotation, sun, stars, geodynamos, galactic dynamos, disk
  LSD: produces fields with spatial coherence , with longterm temporal order (longer than the times-scales of the turbulent motions)

2- Does the dynamo growth depend on Rm? How fast does the dynamo grow?

- Fast dynamo: growth rate can remain finite in the limit Rm $\rightarrow \infty$ (a fast dynamo, whose growth rate does not decrease with decreasing resistivity)
- Slow dynamo: magnetic diffusion is crucial for the operation of the dynamo

III- Questions to ask about any type of physics models?

3- Is it linear or nonlinear?

- Kinematic dynamo: for a given a flow V(x,y,z,t), how fast does the magnetic energy grow? Linear, eigenvalue problem (exponential growth of magnetic field which depends on critical Rm)- **There is no feedback from the Lorentz force.**

$$
\begin{aligned}
\rho(\frac{\partial}{\partial t} - \nu\nabla^2)\mathbf{V} &= -(\mathbf{V}.\nabla)\mathbf{V} + \cancel{(\mathbf{B}.\nabla)\mathbf{B}} - \nabla P + \mathbf{f} \\
(\frac{\partial}{\partial t} - \eta\nabla^2)\mathbf{B} &= -(\mathbf{V}.\nabla)\mathbf{B} + (\mathbf{B}.\nabla)\mathbf{V} \\
\nabla \cdot \mathbf{V} = \nabla \cdot \mathbf{B} &= 0
\end{aligned}
$$

3- Is it linear or nonlinear?

- Kinematic dynamo: for a given a flow V(x,y,z,t), how fast does the magnetic energy grow? Linear, eigenvalue problem (exponential growth of magnetic field which depends on critical Rm)- There is no feedback from the Lorentz force.
- Nonlinear dynamo: nonlinear effects begin to modify the flow to limit further growth of the field. Momentum and induction equations are solved simultaneously.

$$\begin{aligned} \rho\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\mathbf{V} &= -(\mathbf{V}.\nabla)\mathbf{V} + (\mathbf{B}.\nabla)\mathbf{B} - \nabla P + \mathbf{f} \\ \left(\frac{\partial}{\partial t} - \eta\nabla^2\right)\mathbf{B} &= -(\mathbf{V}.\nabla)\mathbf{B} + (\mathbf{B}.\nabla)\mathbf{V} \\ \nabla\cdot\mathbf{V} = \nabla\cdot\mathbf{B} &= 0 \end{aligned}$$

$$\boxed{\mathrm{Rm} = \mathrm{VL}/\eta, \mathrm{Re} = \mathrm{VL}/\nu, \mathrm{Pm} = \mathrm{Rm}/\mathrm{Re}.}$$

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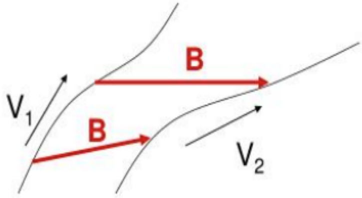
In what dimensional parameter regime is the dynamo operating?

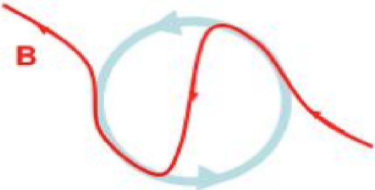
	T [K]	ρ [g cm^{-3}]	P_m	u_{rms} [cm s^{-1}]	L [cm]	R_m
Solar CZ (upper part)	10^4	10^{-6}	10^{-7}	10^6	10^8	10^6
Solar CZ (lower part)	10^6	10^{-1}	10^{-4}	10^4	10^{10}	10^9
Protostellar discs	10^3	10^{-10}	10^{-8}	10^5	10^{12}	10
CV discs and similar	10^4	10^{-7}	10^{-6}	10^5	10^7	10^4
AGN discs	10^7	10^{-5}	10^4	10^5	10^9	10^{11}
Galaxy	10^4	10^{-24}	(10^{11})	10^6	10^{20}	(10^{18})
Galaxy clusters	10^8	10^{-26}	(10^{29})	10^8	10^{23}	(10^{29})

Small Scale Dynamo (SSD)

"Small-scale dynamo" – sustainment of magnetic energy due to magnetic fluctuations in a turbulent state with a small amount of flux. Energy generated by forcing through chaotic flows.

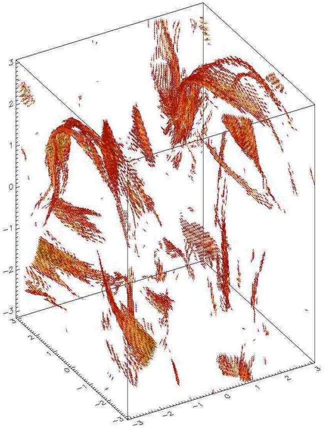
SSD could be non-helical dynamos: amplification of magnetic energy via random walk line stretching, folding, and shear.





Smaller (faster) eddies amplify field (size of eddies are at the resistive scale). **Amplification through random walk in a bath of correlated eddies.**

Small Scale Dynamo (SSD)



SSD: generate magnetic field structures

- extending all the way across the domain
- the orientation of these structures is random folded/stretched intermittent structures and not volume filling (see fig.)
- have a correlation length of order smaller than the forcing scales of the flow.

Kinematic turbulent dynamo

- For a given V(r,t), how would magnetic field grow?
- For example, consider a turbulent velocity field V(r,t) with a spectrum $k^{-5/3}$ (Kolmogorov), would magnetic energy grow out of turbulent eddies?

Kinematic: no back reaction with momentum equation through Lorentz force.

$$\frac{d\mathbf{B}}{dt} = \nabla \times (\mathbf{V} \times \mathbf{B}) - \frac{\eta}{\mu_0} \nabla^2 \mathbf{B}$$

Two main questions:

- what is the critical magnetic Reynolds number for dynamo?
- what is the characteristic scales of the growing magnetic field (spatial structure)?

SSD theory of Kazantsev (1968) predicted that for a non-helical ($< \mathbf{V}.(\nabla \times \mathbf{V}) >= 0$ random (chaotic, zero mean) isotropic, homogeneous flow (delta-correlated in time), dynamo is possible. Simulations needed!Later simulation proved it.

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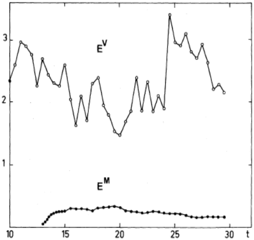
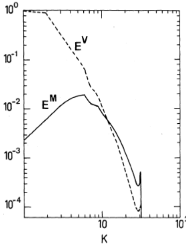
Forced Small Scale Dynamo - Turbulent dynamo

Energies



Spectra



Helical and Nonhelical Turbulent Dynamos

M. Meneguzzi
Centre National de la Recherche Scientifique and Section d'Astrophysique, Division de la Physique, Centre d'Etudes Nucléaires de Saclay, F-91191 Gif-Sur-Yvette, France

and

U. Frisch
Centre National de la Recherche Scientifique, Observatoire de Nice, F-06007 Nice, France

and

A. Pouquet[a]
Centre National de la Recherche Scientifique, Observatoire de Meudon, F-92190 Meudon, France

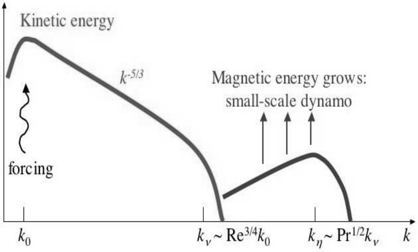
(Received 13 April 1981)

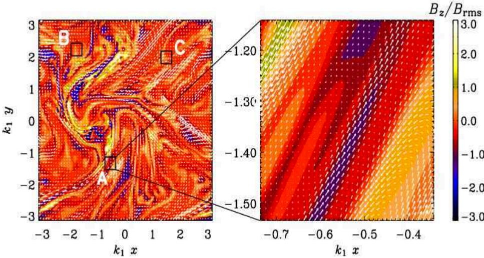
Direct numerical simulations of three-dimensional magnetohydrodynamic turbulence with kinetic and magnetic Reynolds numbers up to 100 are presented. Spatially intermittent magnetic fields are observed in a flow with nonhelical driving. Small-scale helical driving produces strong large-scale nearly force-free magnetic fields.

$$RM = Re = 100$$

- In pioneering work by Meneguzzi, Frisch & Pouquet (1981) full MHD + forcing (feedback through Lorentz force, with a force term **f** to drive flow) were solved.
- Turbulent dynamo with nonhelical driving shown

Forced SSD simulations at high Pm

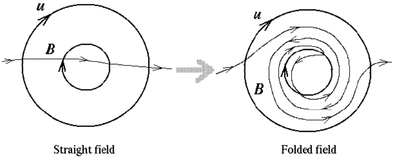




Due to random stretching of the magnetic field by the turbulent motions, folded (intermittent) structures are generated.

[Cattaneo (1996), Schekochihin et al. (2004), Haugen, Brandenburg 2004].

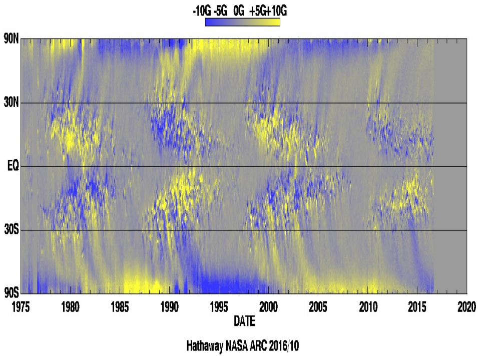
For dynamo action:
stretching should win ($Rm > Rm_{crit}$)
No dynamo:
reconnection wins ($Rm < Rm_{crit}$)



Large-scale dynamo (LSD)

Large-scale dynamo (LSD):

- Dynamo mechanisms that explain the spatial-temporal coherence (regular patterns in space and time which are not random) in astrophysical objects (e. g sun).
- The geometry of the domain and the presence of boundaries and shear are important.

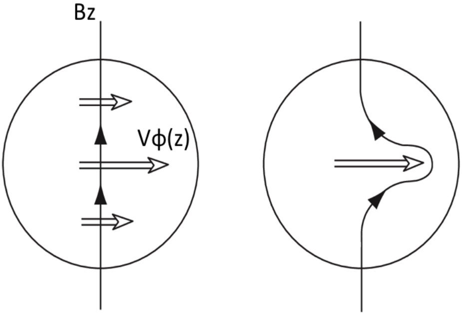


Latitude-time diagram: longitudinally averaged radial component of the observed solar magnetic field
Radial magnetic field alternates in time over the 11 year cycle and also changes sign across the equator.

What is the simplest way that a large-scale field could grow?

Consider: mean seed field + differential rotation

B_0 and $V_\phi(z)$



The Omega-effect:

Conversion of poloidal field to toroidal field by differential rotation (shear) of toroidal flows – (shear of the mean field by differential rotation)

What is the simplest way that a large-scale field could grow?

We go back to the stretching term in the induction equation due to large-scale motions

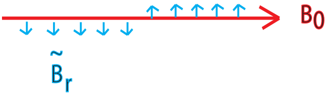
$$\frac{\partial \overline{\mathbf{B}}_{\mathrm{tor}}}{\partial t} = \overline{\mathbf{B}}_{\mathrm{pol}} \cdot \nabla V_{\mathrm{tor}} + \ldots$$

For example consider:
For a poloidal field $B_{pol} = 3 - 10G$ (the upper part of the solar convection zone) how much toroidal field is generated in 3 yrs for an $\Omega_{\odot} = 3 \times 10^{-6} s^{-1}$ (solar angular velocity) and for $\Delta\Omega_{\odot}/\Omega_{\odot} = 0.3$ (a relative latitudinal differential rotation):

$$\Delta B_{\mathrm{tor}} = B_{\mathrm{pol}} \Delta\Omega_{\odot} \Delta T \approx 3 - 10G \times 10^{-6} \times 10^{8} = 0.3 - 1kG$$

so a 1kG toroidal field can be generated in 3 yrs. However this is too weak to rise up coherently all the way from the convection zone, so other dynamo mechanism effects needed

Could a large-scale field grow out of turbulence?



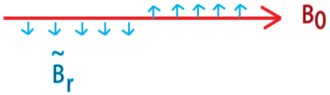
Consider:

$$V = < V > + \widetilde{V}$$
$$B = < B > + \widetilde{B}$$

$\Uparrow$ large-scale $\Uparrow$ small-scale

- fluctuations : $\widetilde{\mathbf{B}}_{mn}(\mathbf{r}) = \widetilde{\mathbf{B}}_{mn}(r)e^{i(m\phi - nz + \delta)} + \mathrm{c.c}$
- $<>$, or overbars $\longrightarrow$ **azimuthally and axially averaged - surface-averaged** $\int d\phi dz$
- Reynold Rules $==> < \widetilde{B} > = < \widetilde{V} >= 0$

Could a large-scale field grow out of turbulence?



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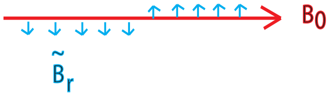

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- Reynold Rules $==> <\widetilde{B}> = <\widetilde{V}> = 0$

Correlated velocity and magnetic field fluctuations give a mean electromotive force (EMF)

Averaging induction equation yields:

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(\overline{\mathbf{V}} \times \overline{\mathbf{B}} + \overline{\mathcal{E}}_{emf} - \eta \nabla \times \overline{\mathbf{B}} \right)$$

where

$$\overline{\mathcal{E}}_{emf} \approx < \widetilde{V} \times \widetilde{B} >$$

is the mean EMF.

Terms only proportional to $\widetilde{V}$ and $\widetilde{B}$ (and the mean) would not contribute to the **mean** induction equation (due to Reynolds rule).

How do we calculate EMFs

Quasilinear theory: Fastest (and easiest) way to find out if any fluctuations would produce **nonzero mean EMF and therefore magnetic field** $==>$

1- linearize MHD equations to find $\widetilde{V}$ and $\widetilde{B}$ and

2- construct EMF $<\widetilde{V}\times\widetilde{B}>$ from linear eigenfunctions to evaluate EMFs

what is the nature of EMFs?

Mean field theory: Finding an expression for the correlator $\overline{\mathcal{E}}_{emf}$ in terms of the mean fields i. e.

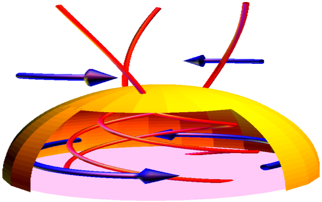
$$\mathcal{E}_i = \alpha_{ij}(\overline{\mathbf{B}}, \Omega, ...)B_i + \eta_{ijk}(\overline{\mathbf{B}}, \Omega, ...)\partial B_j/\partial x_k$$

Tensor components α_{ij} and η_{ijk} are turbulent transport coefficients.

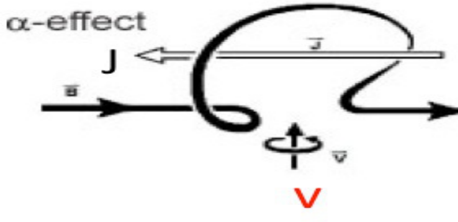
Transport effects that can influence large-scale magnetic fields (Parker, Steenbeck, Krause, Radler, Moffatt).

E. Parker

The Ω and α effects.



from Berger 99



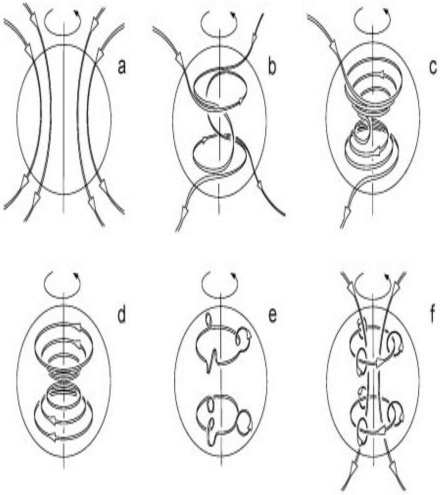
Differential rotation provides a strong source of helicity injection and field growth $\frac{\partial \overline{\mathbf{B}}}{\partial t} \propto \overline{\mathbf{B}}_0 \frac{d}{dz} \overline{\mathbf{V}}(z)$.

$\overline{\mathcal{E}}_{emf} = < \widetilde{V} \times \widetilde{B} > \approx \alpha \overline{\mathbf{B}}$

The electromotive force (EMF) from correlated velocity and magnetic field fluctuations

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{V}} \times \overline{\mathbf{B}} + \alpha \overline{\mathbf{B}} - \eta \nabla \times \overline{\mathbf{B}})$$

Alpha-omega mechanism for modeling of astrophysical rotators



Love, J. J., 1999. Astronomy & Geophysics, 40, 6.14-6.19.

through fluid motions (omega effect)

$\Leftarrow$

$$\frac{\partial \overline{\mathbf{B}}_{\text{tor}}}{\partial t} = \overline{\mathbf{B}}_{\text{pol}} \cdot \nabla V_{\text{tor}} + \ldots$$

through correlated fluctuations(alpha effect)

$$\Leftarrow \frac{\partial \overline{\mathbf{B}}_{\text{pol}}}{\partial t} = \nabla \times \alpha B_{\text{tor}} + \ldots$$

Calculations of transport coefficients

$$\overline{\mathcal{E}}_{emf} =< \widetilde{V} \times \widetilde{B} > =< \widetilde{V} \times \int \frac{\partial \widetilde{B}(t')}{\partial t} dt' > + < \int \frac{\partial (\Delta \widetilde{V})}{\partial t} dt' \times \widetilde{B} >$$

$$= < \int \widetilde{V}(t) \times \nabla \times [\widetilde{V}(t') \times \overline{\mathbf{B}}_0] dt' > + \underbrace{< \int (\widetilde{J} \times \overline{\mathbf{B}}_0) dt' \times \widetilde{B} >}$$

$\Downarrow$

Back-reaction of magnetic field

$$\propto -\widetilde{V}(t) \times (\widetilde{V} \cdot \nabla)\overline{\mathbf{B}} + \widetilde{V}(t) \times (\overline{\mathbf{B}} \cdot \nabla)\widetilde{V}(t)$$

$$\hookrightarrow -\widetilde{V}(t).(\nabla \times \widetilde{V})\overline{\mathbf{B}} + \widetilde{V}^2 \mathbf{J}$$

$$\hookrightarrow = \widetilde{\omega}$$ **(vorticity)**

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Calculations of transport coefficients (cont.)

$$\overline{\mathcal{E}}_{emf} = < \widetilde{V} \times \widetilde{B} > = < \widetilde{V} \times \int \frac{\partial \widetilde{B}(t')}{\partial t} dt' > + < \int \frac{\partial(\Delta \widetilde{V})}{\partial t} dt' \times \widetilde{B} >$$

$$= < \int \widetilde{V}(t) \times \nabla \times [\widetilde{V}(t') \times \overline{\mathbf{B}}_0] dt' > + < \int (\widetilde{J} \times \overline{\mathbf{B}}_0) dt' \times \widetilde{B} >$$

Two correlated signals separated by $t - t'$, the average $< \widetilde{V}(t)\widetilde{V}(t') >= 0$ when $t - t' > \tau_{\rm corr}$, so the integral over time can be approximated with $\tau_{\rm corr}$ for isotropic turbulence, we get:

$$\overline{\mathcal{E}}_{emf} \sim \alpha \overline{\mathbf{B}}_0 - \eta_t \overline{\mathbf{J}}_0$$

where,

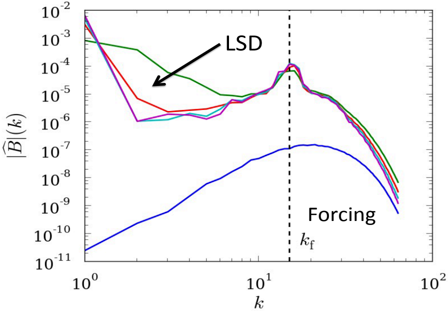
$$\alpha = -\frac{1}{3}\tau_{\rm corr} < \widetilde{V} \cdot \widetilde{\omega} > + \frac{1}{3}\tau_{\rm corr} < \widetilde{J} \cdot \widetilde{B} >$$

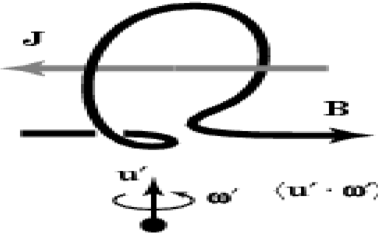
$$\eta_t = \frac{1}{3}\tau_{\rm corr} < \widetilde{V}^2 >$$

Kinetic helicity can give rise to dynamo

$$\alpha = \underbrace{-\frac{1}{3}\tau_{\mathrm{corr}} < \widetilde{V}\cdot\widetilde{\omega} >}_{\alpha_k} + \frac{1}{3}\tau_{\mathrm{corr}} < \widetilde{J}\cdot\widetilde{B} >$$

Helical flows $\rightarrow$ large-scale fields





The uplifting of helical flows with $< \widetilde{V}\cdot\nabla\times\widetilde{V} > \neq 0$ could give an alpha effect.

Back-reaction of the magnetic field modifies the kinematic alpha effect

Back-reaction of the magnetic field on the turbulent flow that generates it (through momentum eq.) modifies the dynamo effect.

$$\alpha = \underbrace{-\frac{1}{3}\tau_{\text{corr}} < \widetilde{V} \cdot \widetilde{\omega} >}_{\alpha_k} + \underbrace{\frac{1}{3}\tau_{\text{corr}} < \widetilde{J} \cdot \widetilde{B} >}_{\alpha_m}$$

The magnetic contribution to the alpha effect with the opposite sign of the kinetic helicity was identified by Pouquet et al. (1976). The term was motivated by the pioneering forced helical nonlinear simulations.

α_m sometimes is called **"current helicity"**, as it is related to magnetic helicity.

Alpha quenching

Using perturbed form of Ohm's law

$$\widetilde{E} = -(\mathbf{V} \times \widetilde{B}) - (\widetilde{V} \times \mathbf{B}) + \eta \widetilde{J} \tag{4}$$

we obtain

$$\boxed{< \widetilde{E} \cdot \widetilde{B} > = < \widetilde{V} \times \widetilde{B} > \cdot \overline{\mathbf{B}} + \eta < \widetilde{J} \cdot \widetilde{B} >} \tag{5}$$

and using

$$\overline{\mathcal{E}}_{emf} \cdot \overline{\mathbf{B}} = < \widetilde{V} \times \widetilde{B} > \cdot \overline{\mathbf{B}} \sim \alpha \overline{\mathbf{B}}^2 - \eta_t \overline{\mathbf{J}}_0 \cdot \overline{\mathbf{B}} \tag{6}$$

and $\alpha = \alpha_k + \alpha_M$ We find an equation for the alpha:

$$\alpha = \frac{\alpha_k + \frac{\tau_{\rm corr}}{3\eta\rho}[\eta_t \overline{\mathbf{J}}_0 \cdot \overline{\mathbf{B}} + < \widetilde{E} \cdot \widetilde{B} >]}{(1 + \frac{\tau_{\rm corr}}{3\eta\rho} B^2)} \tag{7}$$

Gruzinov&Diamond 1994, Bhattacharjee&Yuan 1995

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Alpha quenching

$$\alpha = \frac{\alpha_k + \frac{\tau_{\mathrm{corr}}}{3\eta\rho}[\eta_t \bar{\mathbf{J}}_0 \cdot \bar{\mathbf{B}} + < \tilde{E} \cdot \tilde{B} >]}{(1 + \frac{\tau_{\mathrm{corr}}}{3\eta\rho} B^2)} \quad (8)$$

- When the contribution from the $< \tilde{E} \cdot \tilde{B} >$ is small, and as B increases, α decreases (the so called catastrophic quenching) at high RM (small η).Cattaneo & Hughes 1996, Blackman&Brandenburg 2002
- For example, in solar interior, $Rm = 10^{10}$ could only lead to saturation of $B \approx 10^{-6}$ T which is much weaker that sunspot fields.
- Some contribution from $< \tilde{E} \cdot \tilde{B} >$ could suppress the quenching.
- It has been proposed that **magnetic and current helicity fluxes** could alleviate α quenching.

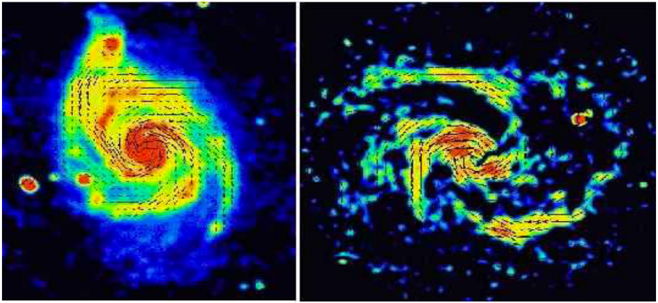
Blackman&Field 2000, Vishniac and Cho 2001

Next Lecture:

- The role of magnetic helicity in the large scale fields
- Theory and simulations link to the observations
- Dynamo in flow-dominated plasmas due to natural instabilities
- Hall dynamos
- Spontaneous plasmoid reconnection

Galactic magnetic fields

Magnetic fields in galaxies are mainly probed using radio observations of their synchrotron emission.



Radio observations of the galaxies M51 and NGC 6946, with superposed magnetic field vectors. Magnetic field (spiral arms) is ordered over large scales. The strength of the total field in the inner spiral arms of M51 is about 30 μG. The strength of field is typically 1...5 μG, and up to $\sim$ 13 μG in the interarm region of NGC 6946.