Turbulence Lecture #4: Understanding magnetized plasma turbulence using experiment & simulation

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Outline

• Measurement and simulation of magnetized (~2D) plasma turbulence characteristics
• Will consider different “flavors” of plasma turbulence that depend on plasma state (R/L_T=-R\nabla T/T, R/L_n, \beta, …)
• Threshold and stiffness of measured and predicted turbulent transport
• Multiscale turbulence characteristics and energy cascades
• Zonal flows & GAMs (Geodesic Acoustic Modes)
• Transport barriers
• Boundary turbulence
Tokamaks

- Axisymmetric
- Helical field lines confine plasma
Tokamaks

- Axisymmetric
- Helical field lines confine plasma
- Closed, nested flux surfaces
We use 1D transport equations to interpret experiments

• Take moments of Vlasov equation + average over short space and time scales of turbulence (assume sufficient scale separation, e.g. $\tau_{turb} << \tau_{transport}$)

• Take flux surface average, i.e. everything depends only on flux surface label ($\rho$) $\rightarrow$ macroscopic transport equation for evolution of equilibrium (non-turbulent) plasma state

\[
\frac{3}{2} n(\rho, t) \frac{\partial T(\rho, t)}{\partial t} + \nabla \cdot Q(\rho, t) = \dot{P}_{source}(\rho, t) - \dot{P}_{sink}(\rho, t)
\]

• To infer experimental transport, $Q_{exp}$:
  – Measure profiles (Thomson Scattering, CHERS)
  – Measure / calculate sources (NBI, RF)
  – Measure / calculate losses (Prad)
Inferred experimental transport larger than collisional (neoclassical) theory – extra “anomalous” contribution

\[ D = -\frac{\Gamma}{\nabla n} \]

\[ \chi = -\frac{Q}{n\nabla T} \]

- Reporting transport as diffusivities – does not mean the transport processes are collisionally diffusive!

Figure 1. Results from TFTR showing ion thermal, momentum, and diffusivities in an L-mode discharge; reprinted with permission from American Institute of Physics.

TFTR
Correlation between local transport and density fluctuations hints at turbulence as source of anomalous transport

Garbet, Nuclear Fusion (1992)
Tynan, PPCF (2009)

\[ \chi = -\frac{Q}{n\nabla T} \]

\[ \frac{\langle \delta n^2 \rangle}{\langle n \rangle^2} \]

\[ Q_{\text{exp}} = Q_{\text{collisions}} + Q_{\text{turbulence}} \]

Our goal is to understand this
We discussed Reynolds stresses $u'_i u'_j$ in Lecture #1. similar concept applies here

- Transport a result of finite average correlation between perturbed drift velocity ($\delta v$) and perturbed fluid moments ($\delta n$, $\delta T$, $\delta v$)
  - Particle flux, $\Gamma = \langle \delta v \delta n \rangle$
  - Heat flux, $Q = 3/2n_0\langle \delta v \delta T \rangle + 3/2T_0\langle \delta v \delta n \rangle$
  - Momentum flux, $\Pi \sim \langle \delta v \delta v \rangle$ (Reynolds stress, just like Navier Stokes)

- Electrostatic turbulence often most relevant $\rightarrow$ $E \times B$ drift from potential perturbations: $\delta v_E = B \times \nabla(\delta \phi)/B^2 \sim k_\theta(\delta \phi)/B$

- Can also have magnetic contributions at high beta, $\delta v_B \sim v_{||}(\delta B_r/B)$ (magnetic “flutter” transport)
40+ years of theory predicts turbulence in magnetized plasma should often be drift wave in nature

General predicted drift wave characteristics (see Lectures #2 & #3):

- Fluctuations in EM fields ($\phi$, B) and fluid quantities (n,v,T) (although really kinetic at high temperature/low collisionality)
- Quasi-2D, elongated along the field lines ($L_{||}>>L_{\perp}$, $k_{||} << k_{\perp}$)
  - Particles can rapidly move along field lines to smooth out perturbations
- Finite-frequency drifting waves, $\omega(k_\theta)\sim\omega_*\sim(k_\theta\rho)v_T/L$
  - Can propagate in ion or electron diamagnetic direction, depending on conditions/dominant gradients
- Perpendicular sizes linked to local gyroradius, $L_{\perp}\sim\rho_{i,e}$ or $k_{\perp}\rho_{i,e}\sim1$
- Correlation times linked to acoustic velocity, $\tau_{cor}\sim c_s/R$
- Fluctuation strength loosely follows mixing length scaling
- In a tokamak expected to be “ballooning”, i.e. stronger on outboard side
  - Due to “bad curvature”/”effective gravity” pointing outwards from symmetry axis
  - Often only measured at one location (e.g. outboard midplane)
Broad drift wave turbulent spectrum verified simultaneously with Langmuir probes and FIR scattering

- Illustrates drift wave dispersion
- However, real frequency almost always dominated by Doppler shift

\[ \omega_{\text{lab}} = \omega_{\text{mode}} (k_\theta) + k_\theta v_{\text{doppler}} \]

- Often challenging to determine mode frequency (in plasma frame) within uncertainties

*FIG. 1. The S(k_\theta, \omega) spectrum at r = 0.255 m in TEXT, from Langmuir probes (contours) and FIR scattering (bars indicate FWHM).*
Small normalized fluctuations in core ($\leq 1\%$) increasing to the edge

- Combination of diagnostics used to measure fluctuation amplitudes

- Measurements also often show $\delta n/n_0 \sim \delta \varphi/T_0$ (electrons nearly Boltzmann)

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**ATF stellarator, Hanson, Nuclear Fusion (1992)**

**TEXT tokamak, Wooton, PoFB (1990)**

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**FIG. 4.** Radial profile of density fluctuations (in %) in ATF stellarator obtained by combining results from different diagnostics [177].

**FIG. 6.** The spatial variation of $\bar{n}/n$ from TEXT ($B_0 = 2$ T, $I_P = 200$ kA, $n_e = 2$ to $3 \times 10^{19}$ m$^{-3}$, H$^+$), shown as crosses (HIBP). Also shown are the predictions of two mixing length estimates, $(\bar{n}/n)^{tor}$ and $(\bar{n}/n)^{slab}$. Both electron feature $\bar{n}/n$ and $k_\phi (k_\phi \rho_e = 0.1)$ are interpreted assuming no ion feature is present.
Mixing length estimate for fluctuation amplitude

- In the presence of an equilibrium gradient, $\nabla n_0$, turbulence with radial correlation $L_r$ will mix regions of high and low density.
Mixing length estimate for fluctuation amplitude

- In the presence of an equilibrium gradient, $\nabla n_0$, turbulence with radial correlation $L_r$ will mix regions of high and low density.
- Leads to fluctuation $\delta n$.
Mixing length estimate for fluctuation amplitude

- In the presence of an equilibrium gradient, \( \nabla n_0 \), turbulence with radial correlation \( L_r \) will mix regions of high and low density.

- Leads to fluctuation \( \delta n \).

- Another interpretation: local, instantaneous gradient limited to equilibrium gradient.
Mixing length estimate for fluctuation amplitude

\[ \delta n \approx \nabla n_0 \cdot L_r \]

\[ \frac{\delta n}{n_0} \approx \frac{\nabla n_0}{n_0} \cdot L_r \approx \frac{L_r}{L_n} \quad (1/L_n = \nabla n_0 / n_0) \]

\[ \frac{\delta n}{n_0} \sim \frac{1}{k_\perp L_n} \sim \frac{\rho_s}{L_n} \quad \left( k_\perp^{-1} \sim L_r; \ k_\perp \rho_s \sim \text{const} \tan t \right) \]

**IF** turbulence scale length linked to \( \rho_s \), would loosely expect \( \delta n/n_0 \sim \rho_s/L_n \)
Fluctuation intensity across machines loosely scales with mixing length estimate, reinforces local $\rho_s$ drift nature.

$Liewer, Nuclear Fusion (1985)$

$\frac{\delta n}{n}$


$log\left(\frac{\delta n}{n}\right)$

$log(\frac{1}{kL_n})$
2D Langmuir probe array in TJ-K stellarator used to directly measure spatial and temporal structures

- Simultaneously acquiring 64 time signals – can directly calculate 2D correlation, with time
- Caveat – relatively cool ($T \sim 10$ eV) compared to fusion performance plasmas ($T \sim 10$ keV)

TJ-K [Ramisch, PoP (2005)]
Radial and poloidal correlation lengths scale with $\rho_s$ reinforcing drift wave nature

- Turbulence close to isotropic
  \[ L_r \sim L_\theta \]

  \[ (L_R \geq L_\theta) \]
Collisionally-excited, Doppler-shifted neutral beam fluorescence

\[ D^0 + e, i \rightarrow \left( D^0 \right)^* \rightarrow D^0 + \gamma(n = 3 \rightarrow 2, \lambda_0 = 656.1 \text{ nm}) \]

**BES Viewing Geometry on DIII-D**

Toroidal Plasma
Neutral Beam
Objective Lens
Optical Fibers

75 KeV \( D^0 \) Neutral Beam (150 L (R))

\[ \frac{\tilde{I}}{\tilde{I} - \mu / n} \]

Spectroscopic imaging provides a 2D picture of turbulence in hot tokamak core: cm spatial scales, μs time scales

- Utilize interaction of neutral atoms with charged particles to measure density

DIII-D tokamak (General Atomics)

Movies at: https://fusion.gat.com/global/BESMovies
BES videos

https://fusion.gat.com/global/BESMovies

(University of Wisconsin; General Atomics)
Many other examples from laboratory experiments illustrating general drift wave expectations

- See supplemental slides for more examples

- General turbulence characteristics are useful for testing theory predictions, but we mostly care about transport
Useful to Fourier decompose transport contributions, especially for theory comparisons

- E.g. particle flux from electrostatic perturbations:

\[
\Gamma(x, t) = \langle \delta n \delta v_r \rangle
\]

\[
\Gamma(k_\theta) = \frac{nT}{B} \sum_{k_\theta} k_\theta \left| \frac{\delta n(k_\theta)}{n_e} \right| \left| \frac{\delta \varphi(k_\theta)}{T_e} \right| \gamma_{n\varphi}(k_\theta) \sin \alpha_{n\varphi}(k_\theta)
\]

- Everything is a function of wavenumber

- Amplitude spectra
  - Coherence
  - Cross phase
Edge Langmuir probe arrays used to decompose turbulent fluxes in $k_\theta$

TJ-K [Birkenmeier, PPCF (2012)]

- Very rare to measure this comprehensively!
- Useful for challenging theory calculations
- Yet to be done this thoroughly for hot tokamak core, where comprehensive gyrokinetic simulations available for comparison
Beyond general characteristics, there are many theoretical “flavors” of drift waves possible in tokamak core & edge

• Usually think of drift waves as gradient driven ($\nabla T_i, \nabla T_e, \nabla n$)
  – Often exhibit threshold in one or more of these parameters

• Different theoretical “flavors” exhibit different parametric dependencies, predicted in various limits, depending on gradients, $T_e/T_i$, $\nu$, $\beta$, geometry, location in plasma…
  – Electrostatic, ion scale ($k_0 \rho_i \leq 1$)
    • Ion temperature gradient (ITG) – driven by $\nabla T_i$, weakened by $\nabla n$
    • Trapped electron mode (TEM) – driven by $\nabla T_e$ & $\nabla n_e$, weakened by $\nu_e$
  – Electrostatic, electron scale ($k_0 \rho_e \leq 1$)
    • Electron temperature gradient (ETG) - driven by $\nabla T_e$, weakened by $\nabla n$
  – Electromagnetic, ion scale ($k_0 \rho_i \leq 1$)
    • Kinetic ballooning mode (KBM) - driven by $\nabla \beta_{pol} \sim \alpha_{MHD}$
    • Microtearing mode (MTM) – driven by $\nabla T_e$, at sufficient $\beta_e$
Challenging to definitively identify a particular theoretical turbulent transport mechanism

• Best we can do:
  – Measure as many turbulence quantities as possible (amplitude spectra, cross-phases, transport)
  – Compare with theory (simulation) predictions
  – Scale equilibrium parameters to investigate trends/sensitivities
  – Make new predictions and test them

• This is the rough outline of “validation” for turbulence and transport theory and modeling

• Let’s look at some modern validation studies
Going to refer to different spatial regions

- Especially **core** (≈100% ionized), **edge** (just inside separatrix), and **scrape-off layer** (SOL, just outside separatrix – open field lines)
CORE ION SCALE TURBULENCE
Transport, density fluctuation amplitude (from reflectometry) and spectral characteristics all consistent with nonlinear ITG simulations in Tore Supra

- Provides confidence in interpretation of transport in conditions when ITG instability/turbulence predicted to be most important

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Casati, PRL (2009)

slope \( \alpha = -4.3 \)

slope \( \alpha = -2.7 \)
Downshift of nonlinear spectra w.r.t. peak linear growth rates

- Linear spectra often peak at higher $k_\theta \rho_s \sim 0.3-1$ than turbulence peaks $k_\theta \rho_s \sim 0.15-0.3 \rightarrow$ indication of inverse cascade from most (linearly) unstable models
Threshold-like behavior observed experimentally

- Experimentally inferred threshold varies with equilibrium, plasma rotation, ...
- Stiffness (~dQ/d∇T above threshold) also varies
- $\chi = -Q/n\nabla T$ highly nonlinear (also use perturbative experiments to probe stiffness)
Threshold like behavior analogous to Rayleigh-Benard instability $\sim \nabla T$

Heat flux $\sim$ heating power

$\text{diffusion} + \text{turbulence}$

$\text{collisional diffusion}$

Temperature gradient
$(T_{\text{hot}} - T_{\text{cold}})$

Analogous to convective transport when heating a fluid from below … boiling water (before the boiling)

Rayleigh, Benard, early 1900's

Threshold gradient for temperature gradient driven instabilities have been characterized over parameter space with gyrokinetic simulations
Modern gyrokinetics has helped uncover key physical effects that influence threshold and stiffness

- Including kinetic electrons, ions, fast ions, realistic equilibrium, collisions, electromagnetic perturbations ($\delta A_\parallel$, $\delta B_\parallel$, drive from rotation gradient, …)

Understanding of thresholds & scaling, and stiffness lead to advances in modeling (Greg’s Lecture #2 & next lecture)
Potentially very important consequence on reactor

Including impact of fast ions + EM effects implies more peaked temperature $\rightarrow$ more fusion power for ITER
Measurement of both electron density and temperature fluctuations at overlapping locations (DIII-D)

- Using electron cyclotron emission (ECE) to measure $\delta T_e$

*DIII-D
White, PoP (2008)*
Normalized density and temperature fluctuations are very similar in amplitude

**DIII-D**
White, PoP (2008)
Comparing $\delta n_e$, $\delta T_e$ fluctuation spectra with simulations using synthetic diagnostic

- Level of agreement sensitive to accounting for realistic instrument function

$\rho=0.5$ (mid-radius)

C. Holland, PoP (2009)
Agreement worse further out ($\rho=0.75$)

- Measured intensity larger than simulations (as is transport), so called “edge shortfall” problem challenging gyrokinetic simulations
Simultaneous measurement of $n_e$ and $T_e$ using same beam path allows for cross-phase measurement
ne-Te cross phases agree amazingly well with simulations!
Measured changes of $\delta T_e$, $n_e-T_e$ crossphase and transport with increasing $\nabla T_e$ provides constraint for simulations

- Increasing fluctuations and transport with $a/L_{Te}$ consistent with enhanced TEM turbulence ($\nabla T_e$ driven TEM, with its own threshold and scaling)

*DIII-D*
*Hillesheim, PRL, PoP (2013)*
Inhomogeneous magnetic field causes trapped particles to precess toroidally.

Trapped electron precession frequencies can be comparable to drift wave frequency \( \omega \sim v_{\parallel}/R \) \( \Rightarrow \) resonance can enhance ITG instability and lead to distinct trapped electron mode (TEM) instabilities driven by \( \nabla T_e, \nabla n_e \).
Simulations can reproduce transport for some observations

- Predicted turbulence levels always too small, even when accounting for sensitivity to $\nabla T_e$
- Discrepancies point to missing physics in theory/simulation – what have we been neglecting?

Holland, PoP (2013)
Instabilities also occur at electron scales

- ITG/TEM turbulence exists at ion scales, $k_{\perp}\rho_i \sim 0.1 - 1$
- ETG instability occurs at electron scales $k_{\perp}\rho_e \sim 0.1 - 1$ ($k_{\perp}\rho_i \sim 6 - 60$)

- ETG is “isomorphic” to ITG
  - Replace $m_i \rightarrow m_e$, $T_i \rightarrow T_e$ so that $\rho_i \rightarrow \rho_e$, $\nu_{Ti} \rightarrow \nu_{Te}$,
  - Replace Boltzmann electrons, $\delta n_e / n_e = e\delta \varphi / T_e$ \rightarrow \text{adiabatic ions, } \delta n_i / n_i = Z_i e\delta \varphi / T_i$

- Recover same threshold and appropriately-normalized growth rate except ETG occurs at scale sizes & times $60 \times$ smaller and faster $[\rho_e / \rho_i \sim (m_e / m_i)^{1/2}$, $V_{Te} / V_{Ti} \sim (m_i / m_e)^{1/2}]$

- Electron scale gyroBohm diffusivity $\left( \chi_{GB,ETG} = \frac{\rho_e^2 V_{Te}}{a} \right)$ also $60 \times$ smaller than ITG, but it can still play an important role…
MULTI-SCALE TURBULENCE
(FROM $\rho_i$ TO $\rho_e$ SCALES)
ETG-like “streamers” predicted to exist on top of ion scale turbulence (https://www.youtube.com/watch?v=3TyHtE9_trg)

Howard, PoP (2014)
Non-intuitive change in predicted transport due to cross-scale coupling between $\sim \rho_i$ and $\sim \rho_e$

- As $a/L_{T_i} \approx -a \nabla T_i / T_i$ is reduced towards ITG threshold, $Q_i$ decreases while electron transport increases due to very small scale ($k_\theta \rho_i > 1$, $k_\theta \rho_e < 1$) turbulence
- Challenge to incorporate in models (next lecture)

Howard, NF (2016)
Hot topic: measure change in turbulence spectrum consistent with multi-scale effects

- Simulations spanning >2 orders of magnitude in both perpendicular dimensions and time (~ 20M cpu-hrs/sim)
- Some “multi-scale” turbulence measurements in L. Schmitz, NF (2012)
In simulations, we can diagnose energy transfer from nonlinear 3-wave interactions

- Nonlinear term is a 3-wave interaction, e.g. \( \delta v \rightarrow v_k \exp(ik \cdot x) \)

\[
(\delta v \cdot \nabla \delta n)_{k_3} \rightarrow \sum_{k_1, k_2} v_{k_1} \cdot k_2 n_{k_2}
\]

where \( k_3 = k_1 + k_2 \)

- Quantify nonlinear energy transfer through the use of bispectra (look for correlated interactions between \( k_1, k_2, k_3 \) satisfying \( k_3 = k_1 + k_2 \))

\[
T(k_3, k_1) = -Re \left\langle \tilde{f}^*(k_3) \tilde{V}_r(k_3 - k_1) \frac{df(k_1)}{dr} \right\rangle \\
- Re \left\langle \tilde{f}^*(k_3) \tilde{V}_\theta(k_3 - k_1) \frac{1}{r} \frac{df(k_1)}{d\theta} \right\rangle,
\]

where \( T(k_3, k_1) \) represents the transfer of energy from gradients of a fluctuating field, \( \tilde{f} \), that exist as a wavenumber, \( k_1 \), to fluctuations existing with a wavenumber, \( k_3 \), which are mediated by \( E \times B \) velocity fluctuations existing with a wavenumber \( k_2 \). The brackets, \( \langle \rangle \) in Equation (1) represent a

Howard, PoP (2016)  
Holland, PoP (2007)  
C. Ritz, PoFB (1989)
Strong electron scale drive and cross-scale coupling leads to significant inverse energy cascade.

Positive: energy transfer from $k_1 \rightarrow k_3$
Strong electron scale drive and cross-scale coupling leads to significant inverse energy cascade

- Low-k turbulence and transport larger than would have been otherwise!

**ETG streamer dominated**

Panel A: $a/L_{T_i} = 1.75$

**ITG dominant**

Panel C: $a/L_{T_i} = 2.25$

Positive: energy transfer from $k_1 \rightarrow k_3$
Strong electron scale drive and cross-scale coupling leads to significant inverse energy cascade

- Low-k turbulence and transport larger than would have been otherwise!
- Also identify more “local-k” inverse cascade (near diagonal)

**ETG streamer dominated**

**ITG dominant**

Positive: energy transfer from $k_1 \rightarrow k_3$
“PURE” ELECTRON SCALE TURBULENCE (not multiscale)
Microwave scattering used to detect high-\(k_{\perp}\) (~mm) fluctuations

- 6 ion radii
- 360 electron radii
- ~2 cm

Mazzucato, PRL (2008)
Smith, RSI (2008)
Guttenfelder, PoP (2011)

NSTX
Applying RF heating to increase $T_e$

Fluctuations increase as expected for ETG turbulence ($R/L_{Te} > R/L_{Te,\text{crit}}$)

- Other trends measured that are consistent with ETG expectations, e.g. reduction of high-$k$ scattering fluctuations with:
  1. Strongly reversed magnetic shear (Yuh, PRL 2011)
     - Simulations predict comparable suppression (Peterson, PoP 2012)
  2. Increasing density gradient (Ren, PRL 2011)
     - Simulations predict comparable trend (Ren, PoP 2012, Guttenfelder NF, 2013, Ruiz PoP 2015)
  3. Sufficiently large $E \times B$ shear (Smith, PRL 2009)
     - Observed in ETG simulations (Roach, PPCF 2009; Guttenfelder, PoP 2011)
Many ETG trends observed in NSTX, challenging to correctly predict transport

• BUT majority of nonlinear gyrokinetic ETG simulations predict $Q_e$ too small to explain experiment

Measured high-k power spectra

Electron heat flux (exp & sim)


Are multi-scale simulations required???
Measurement, simulation and validation has had some success under predominantly electrostatic conditions (low $\beta$)

- But steady state tokamak scenarios require high $\beta$ (for self-generated bootstrap current) → brings in numerous EM effects, has been more challenging to model, is the focus of a lot of recent research
“PURE”
ELECTROMAGNETIC TURBULENCE
Linear microtearing instability

- High-m tearing mode around a rational $q(r_0)=m/n$ surface ($k\cdot B=0 \rightarrow k_{||}(r_0)=0$)
  (Classical tearing mode stable for large $m$, $\Delta'\sim-2m/r<0$)
- In the core, driven by $\nabla T_e$ with time-dependent thermal force $\Rightarrow$ requires collisionality

**Conceptual linear picture**

- Imagine helically resonant ($q=m/n$) $\delta B_r$ perturbation
  $\delta B_r \sim \cos(m\theta - n\phi)$
- $\delta B_r$ leads to radially perturbed field line, finite island width
- $\nabla T_e$ projected onto field line gives parallel gradient
  $\nabla_{||} T_{e0} = \frac{B\cdot \nabla T_{e0}}{B} = \frac{\delta B_r}{B} \nabla T_{e0}$
- Time-dependent parallel thermal force (phase shifted, $\sim i\omega/\nu^* n_e \nabla_{||} T_e$) balanced by inductive electric field $E_{||}=-dA_{||}/dt$ with a $\delta B_r$ that reinforces the instability

- Instability requires sufficient $\nabla T_e$, $\beta$, $\nu_e$ (differences predicted in the edge)
- **Not explicitly driven by bad-curvature**

Onset of magnetic stochasticity leads to large electron thermal transport, $Q_e \sim v_{Te} |\delta B/B|^2$

- Inspecting Poincare plots during early phase of simulation (before saturation)
Microtearing-driven (MT) transport may explain spherical tokamak confinement scaling with collisionality.

\[ \tau_E \sim a^2 / \chi_e \]

![Graph showing experimental data and theoretical predictions](image)

MTM density fluctuations distinct from ballooning modes like ITG (simulations)

NSTX MTM turbulence

DIII-D ITG turbulence
Very challenging to measure small scale internal magnetic fluctuations

- Synthetic diagnostic calculations predict polarimetry could be sensitive

- Cross Polarization Scattering (CPS) may be useful as a local measurement (Rhodes, RSI 2014; Barada, RSI 2017)
Inference of microtearing turbulence via magnetic probes in RFX reversed field pinch (Zuin, PRL 2013)

• Used internal array of closely spaced (~wavenumber resolved) high frequency Mirnov coils (~dB/dt) mounted near vacuum vessel wall

• Confinement and Te increase during “quasi-single helicity” (QSH) state → broadband δB measured (3 below left)

 δB amplitude increases with a/L_{Te} & β (expected for MTM)

• Measured frequency and mode numbers (n,m) align with linear gyrokinetic predictions of MTM

- Additional MTM inferences using novel heavy ion beam probe technique (internal, non-perturbative) in JIPPT-IIU tokamak (Hamada, NF 2015)
ZONAL FLOWS, GAMs

(critical role in saturation of 2D turbulence)
Self-generated “zonal flows” impact saturation of turbulence and overall transport (roughly analogous to jet stream)

- Potential perturbations uniform on flux surfaces, near zero frequency ($f \sim 0$)
- Predator-prey like behavior: turbulence drives ZF (linearly stable), which regulates/clamps turbulence; if turbulence drops enough, ZF drive drops, allows turbulence to grow again…

Linear instability stage demonstrates structure of fastest growing modes

Large flow shear from instability cause perpendicular “zonal flows”

Zonal flows help moderate the turbulence!!!

Rayleigh-Taylor like instability ultimately driving Kelvin-Helmholtz-like instability $\rightarrow$ non-linear saturation
The Jet Stream is a zonal flow (or really, vice-versa)

- NASA/Goddard Space Flight Center Scientific Visualization Studio
Evidence of zonal flows from measuring potential on same flux surface at two different toroidal locations

- High coherency at very low frequency with zero phase shift suggests uniform zonal perturbation
- Also evidence of a coherent mode around 17 kHZ - geodesic acoustic mode ($\omega_{\text{GAM}} \approx c_s/R$) from associated $n=0, m=1$ pressure perturbation

*CHS, Fujisawa, PRL (2004)*
Also found using poloidal flow measurements from BES on DIII-D

- Poloidal flow determined from time delay estimation of poloidally separated BES channels
- High coherency at low frequency, zero phase shift
- Evidence of GAM oscillation
- Relative strength of each varies with radius

_DIII-D, Gupta PRL (2003)_
GAM seen on numerous devices using different measurement techniques

- Seems to be in nearly all machines, if looked for

- See Fig. 11 of Fujisawa, Nuclear Fusion (2009) for legend
Broad cross-machine agreement of GAM frequency with theory

- Discrepancies have spurred additional theory developments to refine gam frequency and damping rates (due to geometry, nonlinear effects, …)

Fujisawa, NF (2009)
EDGE TURBULENCE
H-mode pedestal
Going to refer to different spatial regions in the tokamaks

- Especially **core**, **edge** (just inside separatrix), and **scrape-off layer** (SOL, just outside separatrix)
Spontaneous “H-mode” edge transport barrier can form with sufficient heating power \(\rightarrow\) improved confinement

- Correlated with strong shear in equilibrium radial electric field \(E_r\)
- Suppression of turbulence predicted when equilibrium shearing rate \((\omega_{E\times B})\) > turbulence decorrelation rate \((\Delta \omega_D)\) [Biglari, 1990; Hahm, 1994]

H-mode boundary has nearly collisional transport – like a shear flow boundary layer in neutral fluids
Mean velocity shear flow can suppress turbulence & transport in quasi-2D! (Lecture #1)

- In contrast to flow shear \textit{drive} in 3D turbulence
- Stratospheric ash from Mt. Pinatubo eruption (1991) spread rapidly around equator, \textbf{but confined in latitude by flow shear}

\textbf{Aerosol concentration}

- \textit{Flow shear suppression of turbulence important in magnetized plasmas}
  - See lengthy review by P.W. Terry, Rev. Mod. Physics (2000)
Transition from L→H correlated with drop in turbulence amplitude, reduction in radial correlation length

- Consistent with $E\times B$ shear suppression

- However, there is still no clear understanding regarding what *initiates* the transition and the dynamics involved

- Practically important for understanding how much power required to reach H-mode (*→ almost all reactor designs assume H-mode*)

*Burrell, PoP (1997)*
Local density and magnetic fluctuations measure possible importance of EM turbulence

- Density from reflectometry (& Gas Puff Imaging)
- Magnetic probes inserted 2 cm from separatrix (measures same $k_\theta$ as density)
- Evidence for importance of EM turbulence?
- Leading theory posits KBM (EM drift wave) as a key contributor setting H-mode pedestal (Snyder, NF, 2011)
Various fluctuations observed in ELM free pedestal regions – Weakly Coherent Mode in C-mod I-mode

- I-mode in C-mod similar to H-mode except temperature pedestal only

- Evidence for weakly coherent density, temperature & magnetic fluctuations associated with increased particle transport preventing density pedestal

- Other examples exist in ELM-free H-modes (EHO in DIII-D; QCM in C-Mod)

- Are these weakly coherent, quasi-coherent edge modes really turbulence???
  - Discuss amongst yourselves
Ultimately H-mode edge “pedestal” gradient often close to MHD stability limit

- Kinetic ballooning modes (KBM) are gyrokinetic analog to MHD ballooning modes (see Greg’s Lecture #2, $\alpha_{\text{MHD}}$)
- Considered as ultimate limit on pressure gradient (in the sense of transport, not macroscopic MHD stability)
- One of the most validated models for predicting pedestal pressure relies, in part, on KBM threshold (EPED, Snyder 2000-present)

- More recent simulations suggest other turbulence (MTM, ETG) can be more important (Hatch NF 2015-present)

- This is a very active research area
SCRAPE OFF LAYER TURBULENCE
Going to refer to different spatial regions in the tokamaks

- Especially **core, edge** (just inside separatrix), and **scrape-off layer** (SOL, just outside separatrix – open field lines)

Exhaust heat flux comes out in narrow layer
Understanding scrape-off-layer (SOL) heat-flux width extremely important under reactor conditions

- Narrow SOL heat flux width $\lambda_q$ leads to huge (>10 MW/m$^2$) heat flux density on the divertor plasma facing components (PFCs) $\rightarrow$ significant concern for sputtering and erosion
- Empirical scaling ($\lambda_q \sim 1/B_{\text{pol},\text{MP}}$) very unfavorable for reactors
- Recent turbulence simulations suggest a possible break from this scaling

D. Brunner, APS-DPP (2017)
T. Eich, PRL (2011)

XGC-1 turbulence predictions (C.S. Chang)
Boundary region much harder to diagnose and simulate

- Plasma + neutrals
- Open field lines
- Material boundary conditions
- Boundary turbulence (intermittent blob/filaments vs. gaussian PDF in core)

- See XGC-1, Gkeyll simulations and discussion in Lecture #3
Many options being considered for divertor/SOL magnetic geometry

• Requires additional complexity in poloidal field coils and controllability
• Generally will also required impurity seeding in core/edge plasma to radiate much of the power
• Spreading (from turbulence) could reduce heat flux density

X divertor

Snowflake divertor

Super-X divertor

X Divertor

DIII-D

NSTX-U

MAST-U

\[ T2 \sim 2.3 \, m^2 \]

\[ T1 \sim 0.78 \, m^2 \]
Edge Turbulence Measurements in NSTX

- High speed cameras make images of edge turbulence
- 3-D ‘filaments’ localized to 2-D by gas puff imaging (GPI)

Lots of videos via Stewart Zweben:
http://w3.pppl.gov/~szweben/

• This movie 285,000 frames/sec for ~ 1.4 msec
• Viewing area ~ 25 cm radially x 25 cm poloidally

L-H mode transition t~0.245 s
Outside separatrix, blobs can be ejected and self-propagate to vessel wall

- Plasma is much less dense farther out in scrape-off layer
- Relative intensity of blob becomes large (\( \delta I/I \))
- Distribution of perturbations becomes strongly non-Gaussian (→ intermittent)
Theories and simulations exist that predict blob characteristics: size, density, velocity

- Fluid theories for lower temperature edge have provided some insight on predicted blob sizes and propagation velocities
Filaments seen on bad-curvature side of divertor legs

- Distributed over range of toroidal mode numbers (k) and frequency

C III emission imaging divertor “legs” ($T_e \sim 5-10$ eV)

Filaments on divertor legs are decorrelated from upstream (midplane) blobs – disconnection due to strong magnetic shear around X-point? New source of instability drive?

Scotti (2018)
Summary

• Although challenging to measure, a number of plasma turbulence characteristics have been measured and used to help validate and improve theoretical predictions and understanding.

• With this improved understanding we desire reduced models that predict turbulence and the transport it causes → next lecture.
EXTRA SLIDES
Gyrokinetics in brief – evolving 5D gyro-averaged distribution function

\[
\frac{\omega}{\Omega} \ll 1
\]

\[
f(\bar{x}, \bar{v}, t) \xrightarrow{\text{gyroaverage}} f(\bar{R}, v_\parallel, v_\perp, t)
\]

- Average over fast gyro-motion → evolve a distribution of gyro-rings

Gyrokinetics in brief – evolving 5D gyro-averaged distribution function

\[ \frac{\omega}{\Omega} \frac{\rho}{L} \frac{\delta f}{f_0} \frac{k_{||}}{k_{\perp}} \ll 1 \]

\[ f(\vec{x}, \vec{v}, t) \xrightarrow{\text{gyroaverage}} f(\vec{R}, v_{||}, v_{\perp}, t) \]

\[ f = F_M + \delta f \]

\[ \frac{\partial (\delta f)}{\partial t} + v_{||} \hat{b} \cdot \nabla \delta f + \delta v_d \cdot \nabla \delta f + \delta \vec{v} \cdot \nabla F_M + \vec{v}_{E_0}(r) \cdot \nabla \delta f + \delta \vec{v} \cdot \nabla \delta f = C(\delta f) \]

- Fast parallel motion
- Slow perpendicular toroidal drifts
- Advection across equilibrium gradients (\(\nabla T_0, \nabla n_0, \nabla v_0\))
- Dopper shift due to sheared equilibrium \(E_\ell(r)\)
- Collisions

\[ \vec{v}_{\perp} = \frac{m v_{||}^2 \hat{b} \times \hat{k}}{qB} \]

\[ \vec{v}_{\parallel B} = \frac{m v_{\perp}^2 \hat{b} \times \nabla B / B}{2} qB \]

\[ \delta v_a = \frac{c}{B} \hat{b} \times \nabla \Psi_a \]

\[ \Psi_a(\vec{R}) \doteq \left. \left\langle \delta \phi(\vec{R} + \rho) - \frac{1}{c}(\vec{V}_0 + \vec{v}) \cdot \delta A(\vec{R} + \rho) \right\rangle \right|_R \]

- Must also solve gyrokinetic Maxwell equations self-consistently to obtain \(\delta \varphi, \delta B\)

NSTX-U

Guttenfelder – UCLA Plasma Seminar (Feb. 11, 2016)
Why does turbulence develop in tokamaks?

**Example:** Linear stability analysis of Ion Temperature Gradient (ITG) “ballooning” micro-instability (expected to dominate in ITER)
Toroidicity Leads To Inhomogeneity in $|\mathbf{B}|$, gives $\nabla \mathbf{B}$ and curvature ($\kappa$) drifts

$\vec{V}_\kappa = \frac{m v^2}{qB} \hat{b} \times \mathbf{k} \sim T_\parallel$

$\vec{V}_{\nabla \mathbf{B}} = \frac{m v^2}{2} \frac{\hat{b} \times \nabla \mathbf{B}/B}{qB} \sim T_\perp$

- What happens when there are small perturbations in $T_\parallel$, $T_\perp$? $\Rightarrow$ Linear stability analysis...
Temperature perturbation ($\delta T$) leads to compression ($\nabla \cdot v_{di}$), density perturbation – $90^\circ$ out-of-phase with $\delta T$

- Fourier decompose perturbations in space ($k_0 \rho_i \leq 1$)
- Assume small $\delta T$ perturbation
Dynamics Must Satisfy Quasi-neutrality

- Quasi-neutrality (Poisson equation, \(k_i \ll c < 1\)) requires

\[
\nabla^2 \tilde{\phi} = \frac{1}{\varepsilon_0} \sum_s e Z_s \int d^3 v f_s
\]

\[
\left( k^2 \lambda_D^2 \right) \frac{\tilde{\phi}}{T} = \frac{\tilde{n}_i - \tilde{n}_e}{n_0}
\]

- For this ion drift wave instability, parallel electron motion is very rapid

\[\mathbf{v}_e \ll \mathbf{v}_{Te}\] maintain a Boltzmann distribution

\[
0 = -T_e \nabla \tilde{n}_e + n_e e \nabla \tilde{\phi}
\]

\[
\omega < k || v_{Te} \rightarrow n_0 + \tilde{n}_e = n_0 \exp(e \tilde{\phi} / T_e)
\]

\[
\tilde{n}_e \approx n_0 e \tilde{\phi} / T_e \Rightarrow \tilde{n}_e \approx \tilde{\phi}
\]
Perturbed Potential Creates $E \times B$ Advection

- Advection occurs in the radial direction

$\hat{b} \times \nabla B$ – ions

$\nabla B$, curvature

$\vec{v}_{d,ion}$

$\sim$Boltzmann e’s

$T^+$

$T^-$

$T^+$

$T^-$

$T^+$

$T^-$

$\nabla T$

$\nabla n$

$E_q$

$E_\theta$

$E_\theta$

$E_\theta$

$E_\theta$
Background Temperature Gradient Reinforces Perturbation $\rightarrow$ Instability

\[ \nabla T \]

\[ \vec{v}_{E \times B} = \frac{\vec{E} \times \hat{b}}{B} \]
Analogy for turbulence in tokamaks – Raylor-Taylor instability

- Higher density on top of lower density, with gravity acting downwards.
Inertial force in toroidal field acts like an effective gravity.

Unstable in the outer region.

GYRO code
https://fusion.gat.com/theory/Gyro
Same Dynamics Occur On Inboard Side But Now Temperature Gradient Is Stabilizing

- Advection with $\nabla T$ counteracts perturbations on inboard side – “good” curvature region

\[ \vec{v}_{E \times B} = \frac{\vec{E} \times \hat{b}}{B} \]
Fast Parallel Motion Along Helical Field Line Connects Good & Bad Curvature Regions

- Approximate growth rate on outboard side
- Parallel transit time
- Expect instability if $\gamma_{\text{instability}} > \gamma_{\text{parallel}}$, or

\[ \gamma_{\text{instability}} \sim \frac{V_{\text{th}}}{\sqrt{RL_T}} \quad 1/L_T = -1/T \cdot \nabla T \]

\[ \gamma_{\text{parallel}} \sim \frac{V_{\text{th}}}{qR} \]

\[ \left( \frac{R}{L_T} \right)_{\text{threshold}} \approx \frac{1}{q^2} \]
Threshold like behavior analogous to Rayleigh-Benard instability

Threshold gradient for temperature gradient driven instabilities have been characterized over parameter space with gyrokinetic simulations.

Analogous to convective transport when heating a fluid from below … boiling water (before the boiling).

Temperature gradient

\[ T_{\text{hot}} - T_{\text{cold}} \]

Heat flux $\sim$ heating power

diffusion + turbulence

collisional diffusion

Rayleigh, Benard, early 1900's
Finite gyroradius effects limit characteristic size to ion-gyroradius \((k_\perp \rho_i \sim 1)\)

- Microinstabilities depend on \(E \times B\) drift from potential perturbations
  
  \[ \vec{v}_E = \frac{\hat{b} \times \nabla \phi}{B} = -i k_\perp \frac{\phi}{B} = -i k_\perp \left( \frac{\phi}{T_i} \right) \left( \frac{T_i}{B} \right) = -i (k_\perp \rho_i) \left( \frac{\phi}{T_i} \right) v_{Ti} \]

- Normalized amplitude increases as \( (v_E/v_{Ti}) \sim (k_\perp \rho_i)(\phi/T_i) \rightarrow \) drift velocity increases with smaller wavelength (larger \(k_\perp \rho_i\))
  
  – For small \((k_\perp \rho_i) << 1\), growth rates increase linearly with \(~(k_\perp \rho_i)\)

- If wavelength approaches ion gyroradius \((k_\perp \rho_i) \geq 1\), average electric field experienced over fast ion-gyromotion is reduced:
  
  \[ \langle \nabla \phi \rangle_{\text{gyro-average}} \sim \nabla \phi \quad \langle \nabla \phi \rangle_{\text{gyro-average}} \sim \nabla \phi [1-(k_\perp \rho_i)^2] \]

\(\Rightarrow\) Maximum growth rates (and typical turbulence scale sizes) occur for \((k_\perp \rho_i) \sim 1\)
Ion scales ($k_{\perp}\rho_i \sim 1$)

- Ion temperature gradient ($\text{ITG}, \gamma \sim \nabla T_i$) via ion compressibility ($\sim \nabla B, \kappa$)
- Trapped electron mode ($\text{TEM}, \gamma \sim \nabla T_e, \nabla n_e$) from electron trapping ($\sim f_t$)

Electron scales ($k_{\perp}\rho_e \sim 1$)

- Electron temperature gradient ($\text{ETG}, \gamma \sim \nabla T_e$), analogous to ITG ($\sim \nabla B, \kappa$)

- Instabilities driven by gradients ($\nabla T_i, \nabla T_e, \nabla n$) surpassing thresholds which depend on: connection length ($\sim qR$), magnetic shear ($dq/dr$), temperature ratio ($T_e/T_i$), additional equilibrium effects …
Aspect ratio is an important free parameter, can try to make smaller reactors (i.e. cheaper)

\[
\text{Aspect ratio } A = \frac{R}{a}
\]
\[
\text{Elongation } \kappa = \frac{b}{a}
\]

But smaller \( R \) = larger curvature, \( \nabla B \sim \frac{1}{R} \) -- isn’t this terrible for “bad curvature” driven instabilities?!?!?!
Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

- Short connection length → **smaller average bad curvature**
Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

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- Quasi-isodynamic (~constant B) at high $\beta$ → **grad-B drifts stabilizing** [Peng & Strickler, NF 1986]
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- Strong coupling to $\delta B_\perp \sim \delta A_\parallel$ at high β → **stabilizing to ES-ITG**

\[ \text{ITG growth rate} \]

\[ \beta \]

Kim, Horton, Dong, PoFB (1993)
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- Strong coupling to $\delta B_{\perp} \sim \delta A_{\parallel}$ at high $\beta$ $\rightarrow$ **stabilizing to ES-ITG**
- Small inertia ($n m R^2$) with uni-directional NBI heating gives strong toroidal flow & flow shear $\rightarrow$ **E×B shear stabilization** ($dv_{\perp}/dr$)

Biglari, Diamond, Terry, PoFB (1990)
Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

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- Quasi-isodynamic ($\sim$constant B) at high $\beta$ $\rightarrow$ grad-B drifts stabilizing [Peng & Strickler, NF 1986]
- Large fraction of trapped electrons, BUT precession weaker at low $A$ $\rightarrow$ reduced TEM drive [Rewoldt, Phys. Plasmas 1996]
- Strong coupling to $\delta B_\perp \sim \delta A_\parallel$ at high $\beta$ $\rightarrow$ stabilizing to ES-ITG
- Small inertia (nmR$^2$) with uni-directional NBI heating gives strong toroidal flow & flow shear $\rightarrow$ $E\times B$ shear stabilization ($dv_\perp/dr$)

$\Rightarrow$ Not expecting strong ES ITG/TEM instability (much higher thresholds)

- BUT
- High beta drives EM instabilities: microtearing modes (MTM) $\sim \beta_e \cdot \nabla T_e$, kinetic ballooning modes (KBM) $\sim \alpha_{MHD} \sim q^2 \nabla P/B^2$
- Large shear in parallel velocity can drive Kelvin-Helmholtz-like instability $\sim dv_\parallel/dr$
Ion thermal transport in H-modes (higher beta) usually very close to collisional (neoclassical) transport theory

- Consistent with ITG/TEM stabilization by equilibrium configuration & strong $E \times B$ flow shear
  - Impurity transport (intrinsic carbon, injected Ne, …) also usually well described by neoclassical theory [Delgado-Aparicio, NF 2009 & 2011 ; Scotti, NF 2013]

- **Electron energy transport always anomalous**
  - Toroidal angular momentum transport also anomalous (Kaye, NF 2009)
Predicted dominant core-gradient instability correlated with local beta and collisionality

- For sufficiently small $\beta$, ES instabilities can still exist (ITG, TEM, ETG)
- At increasing $\beta$, MTM and KBM are predicted $\rightarrow$ depending on $\nu$
  - Various instabilities often predicted in the same discharge – global, nonlinear EM theory & predictions will hopefully simplify interpretation (under development)


Local gyrokinetic analyses at $\sim$2/3 radius

Guttenfelder, NF (2013)

NSTX