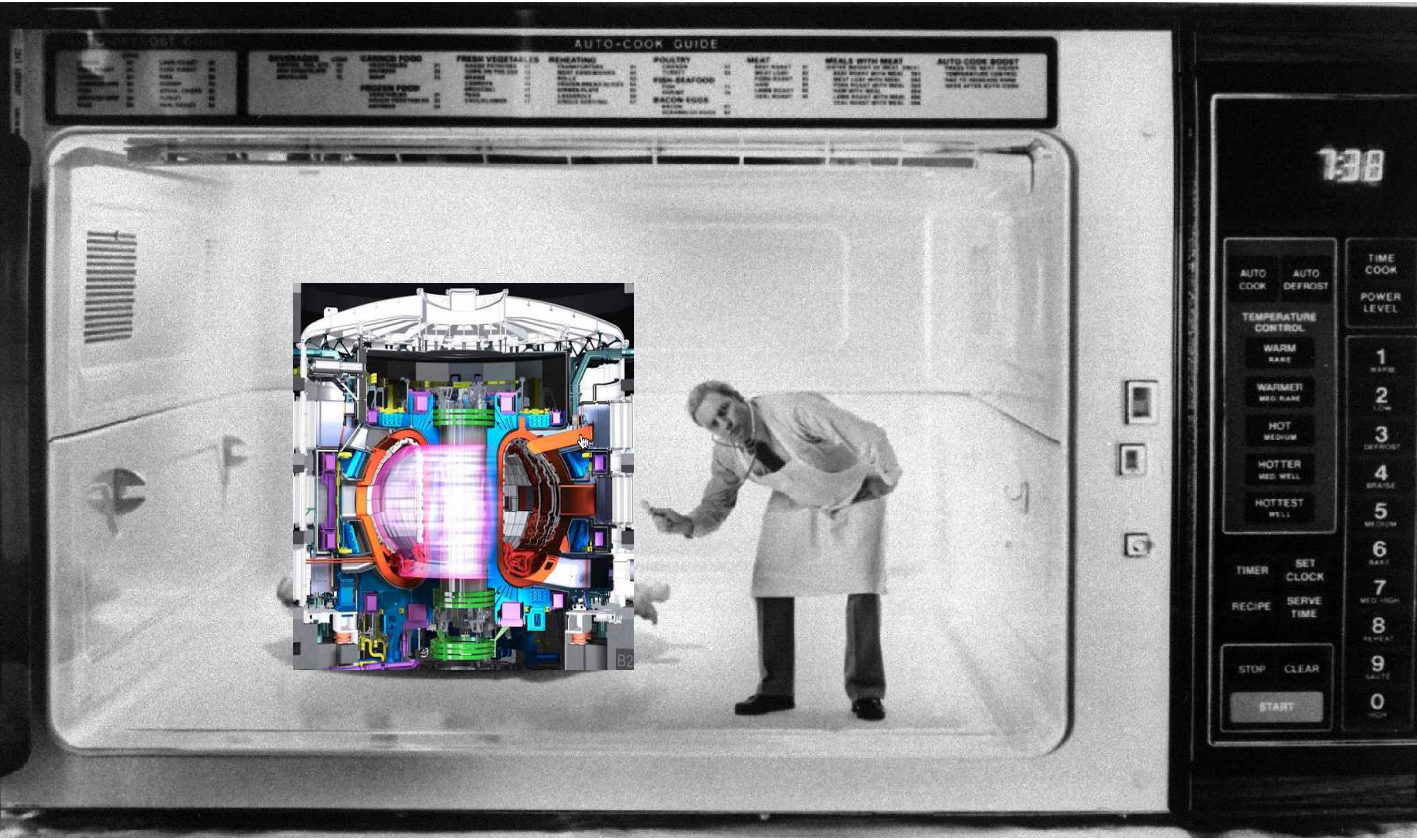
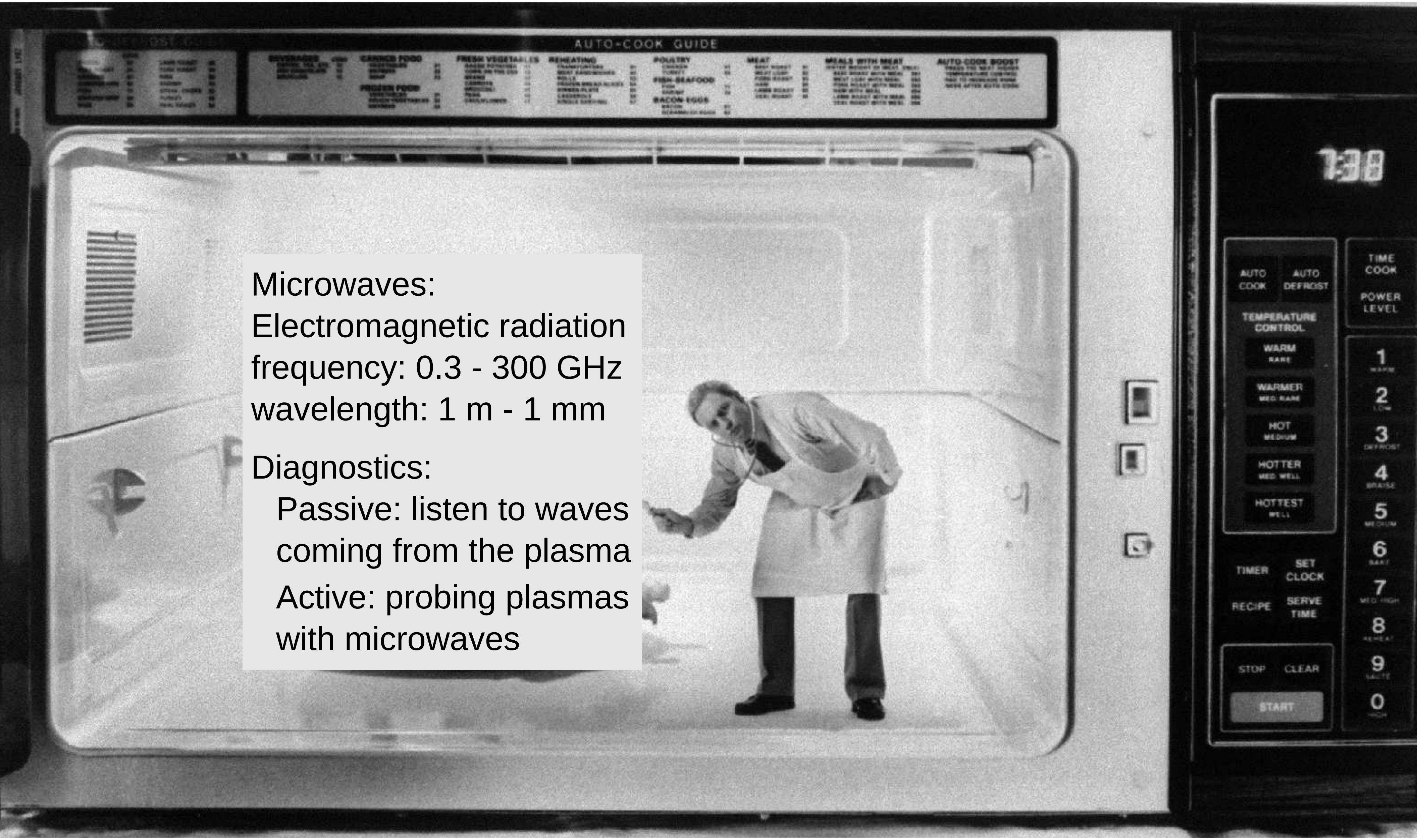
Microwave Diagnostics for Plasmas

Gerrit J. Kramer, PPPL



Microwave Diagnostics for Plasmas



Microwaves:
Electromagnetic radiation
frequency: 0.3 - 300 GHz
wavelength: 1 m - 1 mm

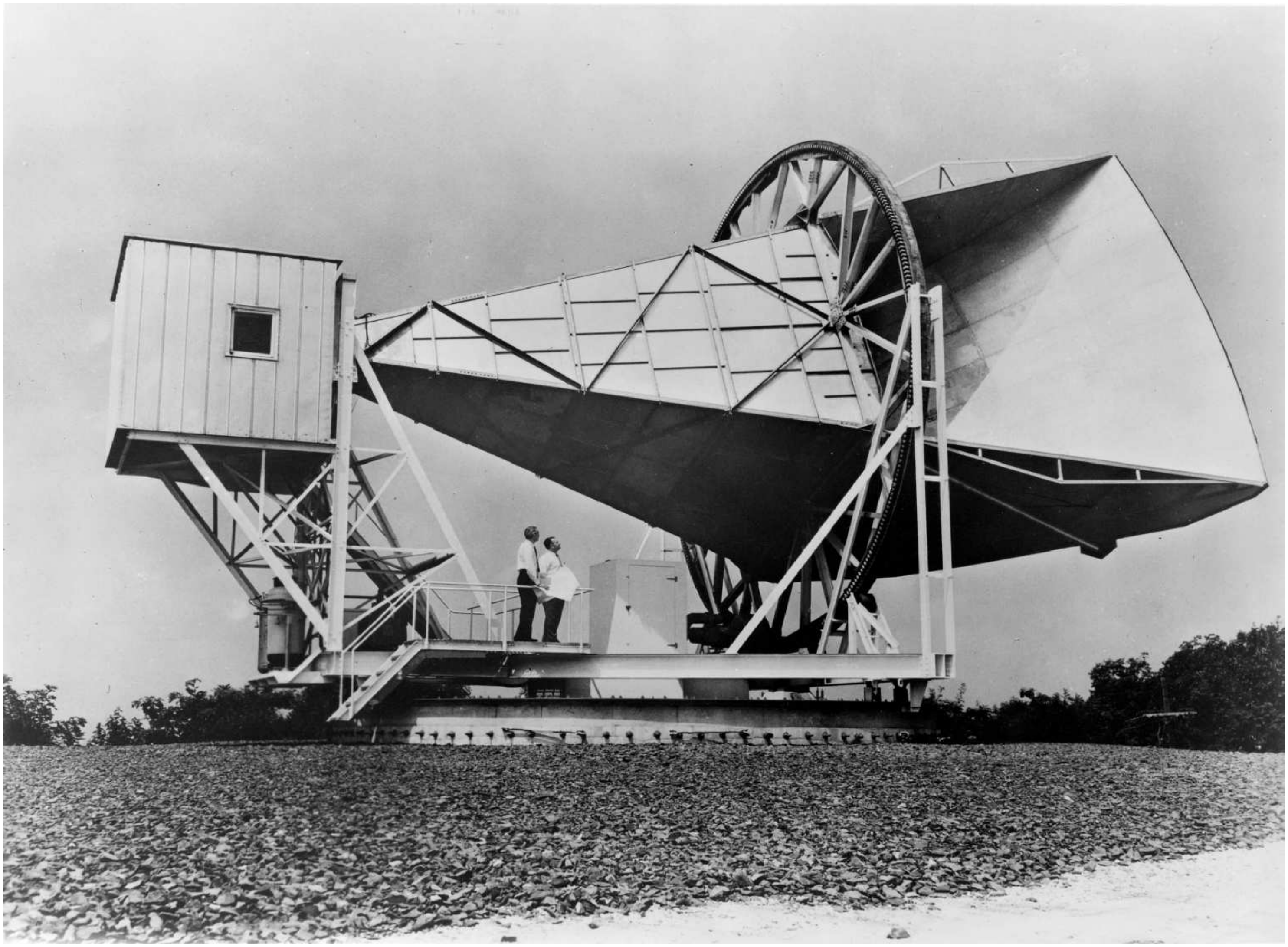
Diagnostics:
- Passive: listen to waves coming from the plasma
- Active: probing plasmas with microwaves

Listening to the oldest plasma signal in the universe: A noise problem in a microwave antenna

- Arno Penzias and Robert W. Wilson found unexplained noise signal in their microwave antenna

- Robert Dicke, Jim Peebles, Peter Roll, and David Wilkinson recognized this signal as coming from the early universe

- This "noise" is now know as:
  Cosmic
  Microwave
  Background
  (CMB) radiation



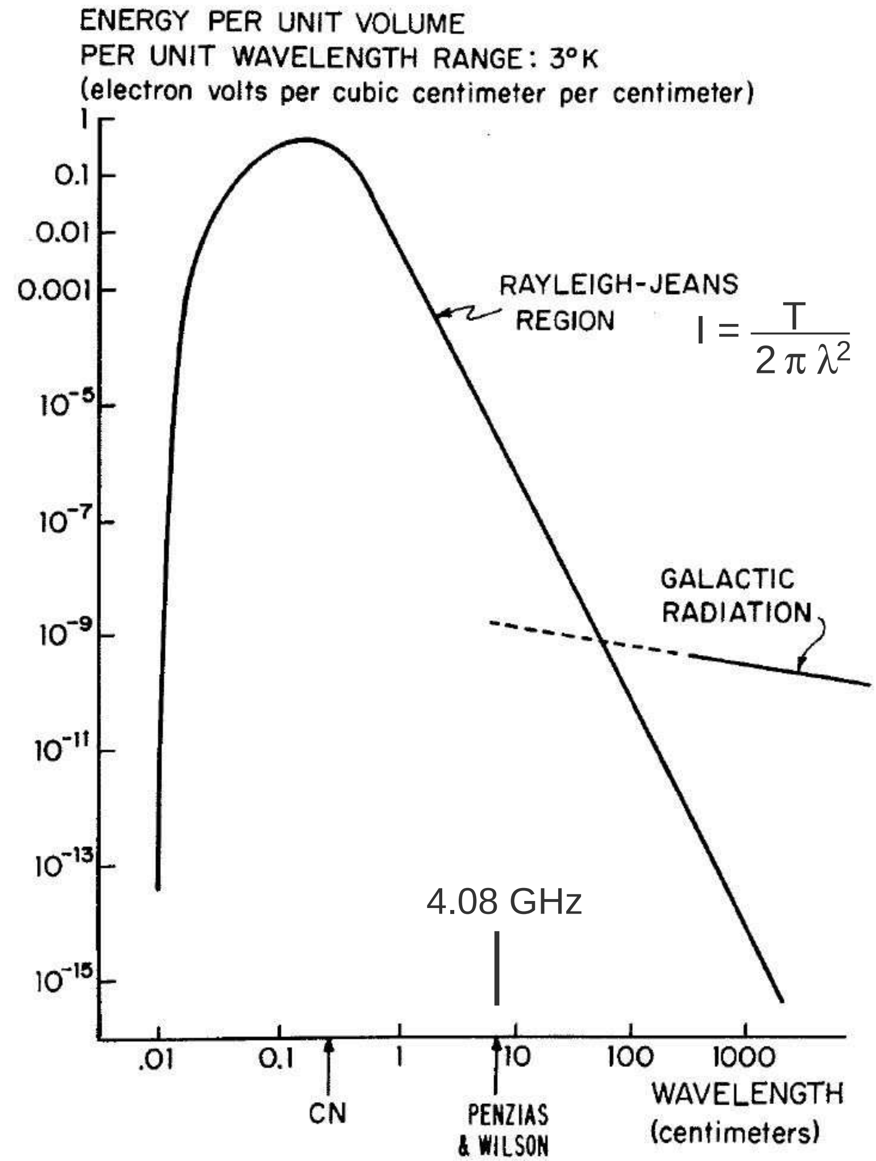
Listening to the oldest plasma signal in the universe: How the cosmic microwave background is formed

- After its formation, the early universe was a hot and rapidly expanding plasma consisting of:
  photons, electrons, and protons
  in thermal equilibrium

- About 380000 years after the Big Bang the temperature dropped to ~3000 K when protons and electrons combined to hydrogen

- The photons could no longer interact with the electrons and therefore, the thermal equilibrium between matter and photons was lost ant the universe became transperant for fotons

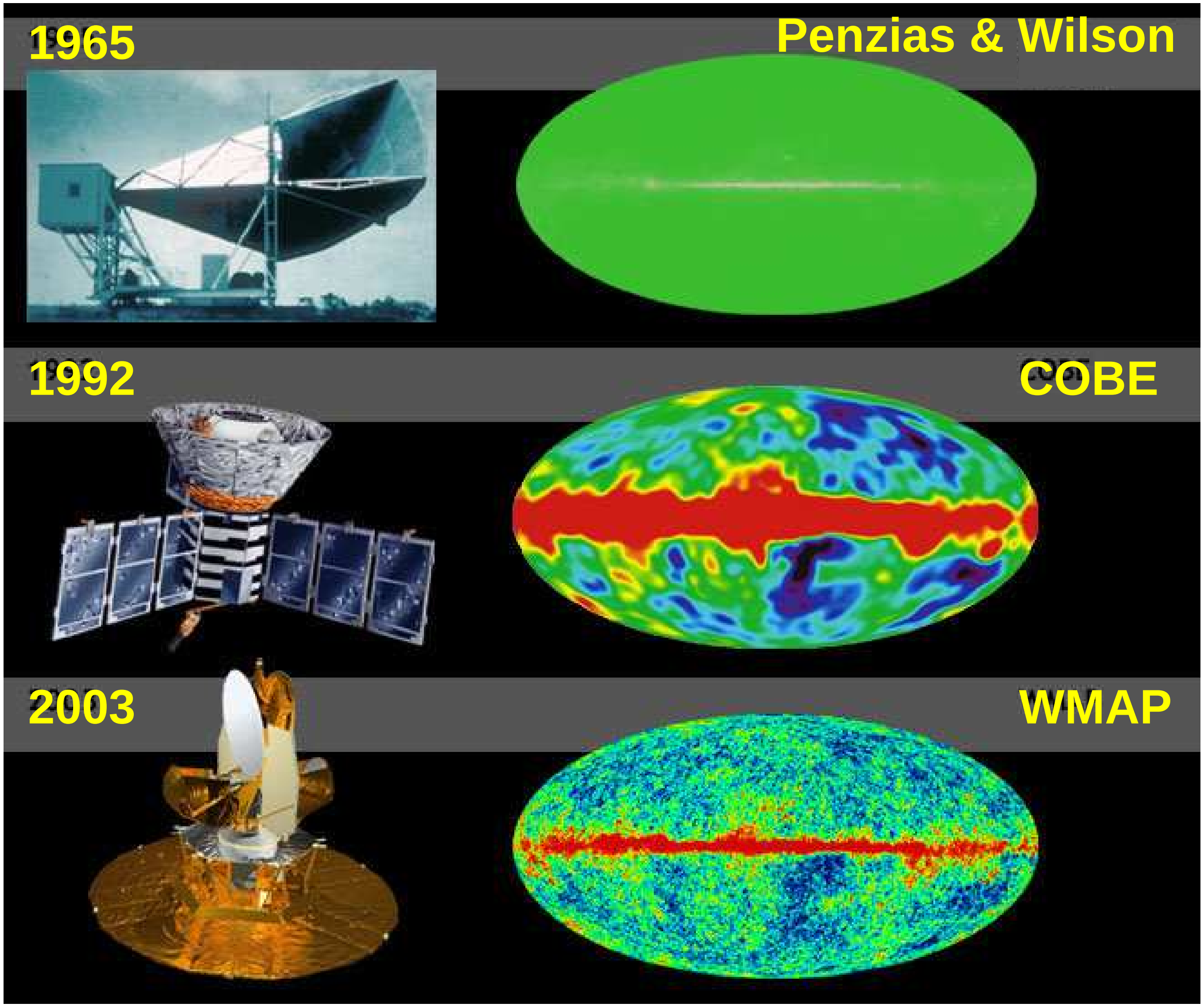
- Since the time that the photons left thermal equilibrium the universe has expanded ~1100 times and the photon temperature has dropped to: 3000 / 1100 = 2.7 K



From: S. Weinberg, The first three minutes

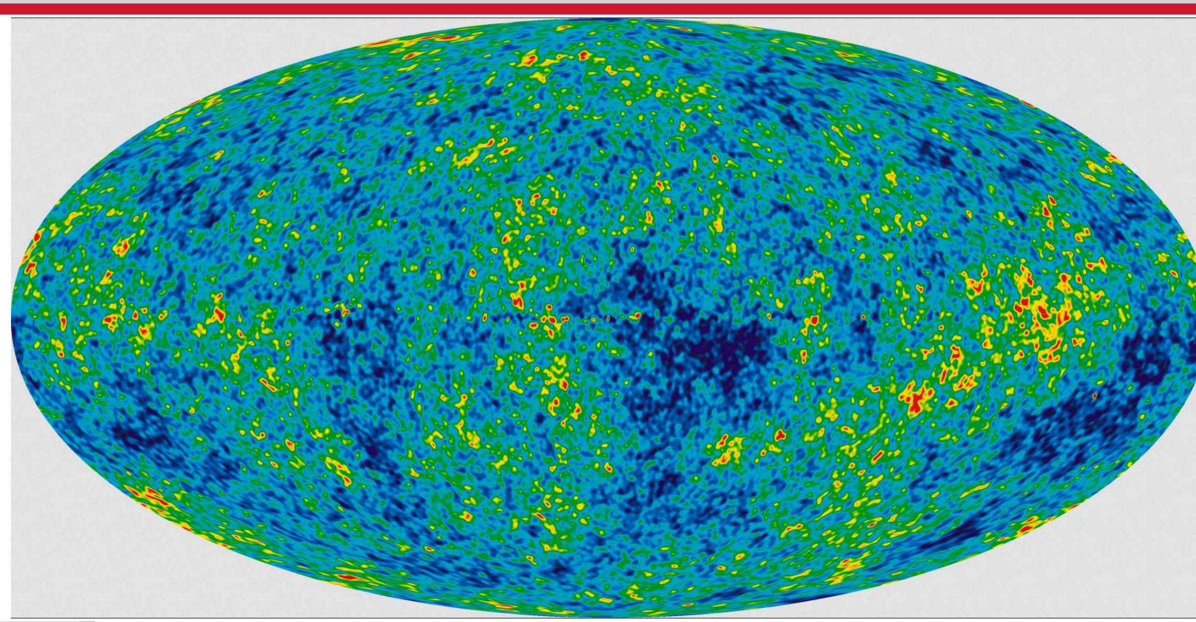
Listening to the oldest plasma signal in the universe: Detector improvements lead to better CMB maps

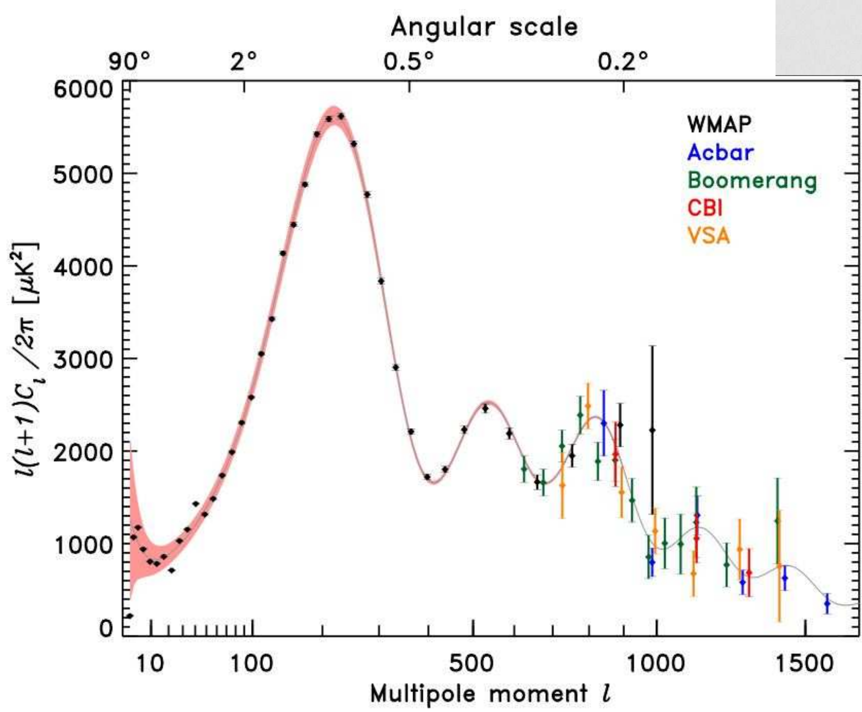
- Small variations in the cosmic microwave background temperature reveal density variations in the primodial plasma
- These density variations became the seeds for galaxies



Listening to the oldest plasma signal in the universe: The CMB map guides understanding of the universe

- After subtracting the radiation from the Milky way the data is used for testing cosmological models





Best-fit cosmological parameters from WMAP nine-year results[14]

Parameter	Symbol	Best fit (WMAP only)	Best fit (WMAP + eCMB + BAO + H_0)
Age of the universe (Ga)	t_0	13.74 ±0.11	13.772 ±0.059
Hubble's constant ($^{km}/_{Mpc \cdot s}$)	H_0	70.0 ±2.2	69.32 ±0.80
Baryon density	Ω_b	0.0463 ±0.0024	0.046 28 ±0.000 93
Physical baryon density	$\Omega_b h^2$	0.022 64 ±0.000 50	0.022 23 ±0.000 33
Cold dark matter density	Ω_c	0.233 ±0.023	$0.2402^{+0.0088}_{-0.0087}$
Physical cold dark matter density	$\Omega_c h^2$	0.1138 ±0.0045	0.1153 ±0.0019
Dark energy density	Ω_Λ	0.721 ±0.025	$0.7135^{+0.0095}_{-0.0096}$
Density fluctuations at $8h^{-1}$ Mpc	σ_8	0.821 ±0.023	$0.820^{+0.013}_{-0.014}$
Scalar spectral index	n_s	0.972 ±0.013	0.9608 ±0.0080
Reionization optical depth	τ	0.089 ±0.014	0.081 ±0.012
Curvature	$1-\Omega_{\mathrm{tot}}$	$-0.037^{+0.044}_{-0.042}$	$-0.0027^{+0.0039}_{-0.0038}$
Tensor-to-scalar ratio (k_0 = 0.002 Mpc^{-1})	r	< 0.38 (95% CL)	< 0.13 (95% CL)
Running scalar spectral index	$dn_s/d\ln k$	−0.019 ±0.025	−0.023 ±0.011

[14] Bennett, C. L.; et al. (2013), Astrophysical Journal Supplement, 208 20

Temperature measurements of magnitized electrons electron cyclotron emission

- Charged particles in a magnetic field follow spiraling orbits due to the Lorents Force:

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

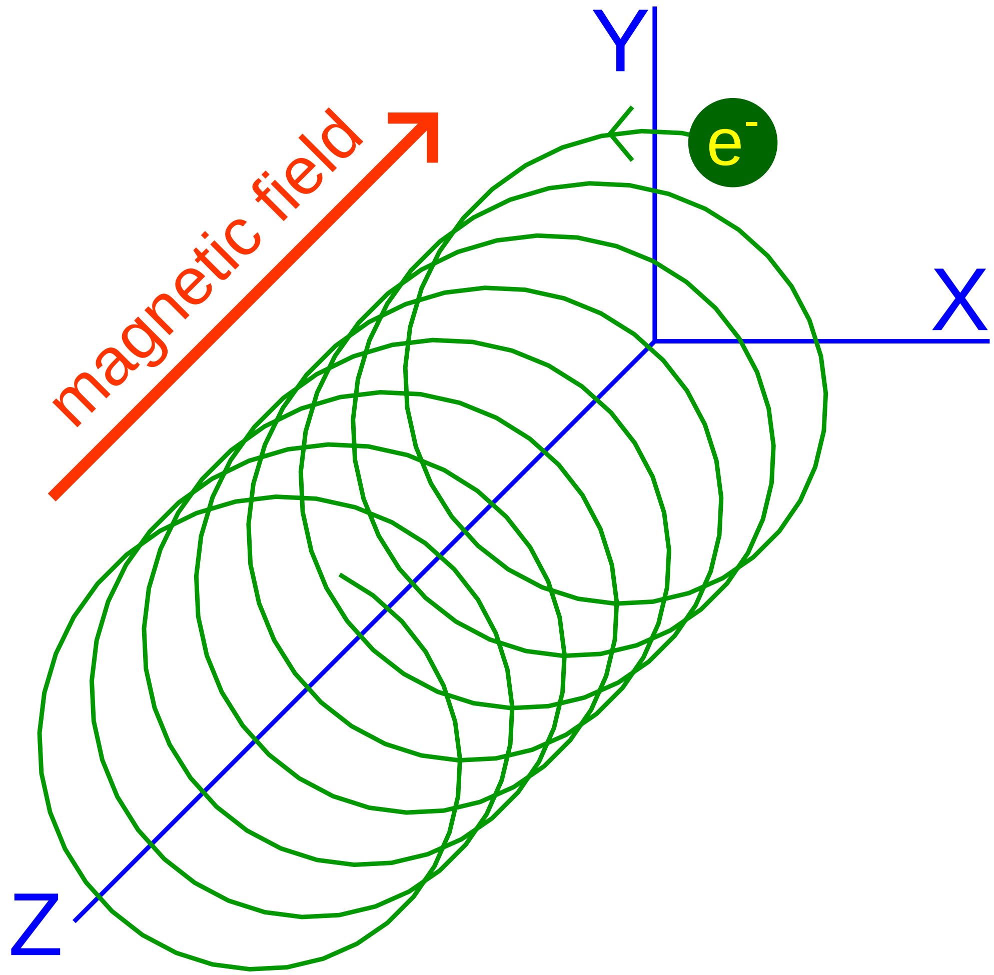
- Accelerating electrons emit radiation: Electron Cyclotron Emission or ECE

- solution of Lorentz equation:

$$\frac{d\mathbf{R}}{dt^2} = \frac{q}{m}\frac{d\mathbf{R}}{dt} \times \mathbf{B}$$

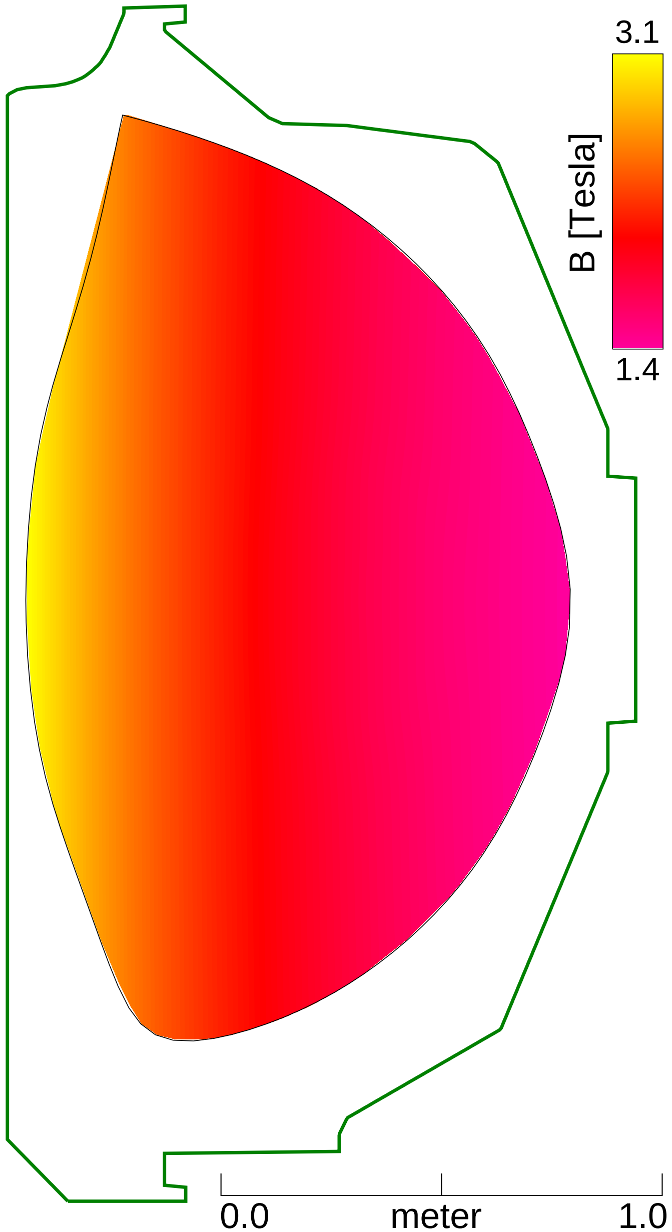
$\mathbf{B} = (0,0,B)$ and $\mathbf{v} = (v_\perp, v_{\parallel})$

$$R_x = \rho \sin(\omega t) \qquad \rho = \frac{v_\perp}{\omega}$$
$$R_y = \rho \sin(\omega t)$$
$$R_z = v_{\parallel} t \qquad \omega_c = \frac{qB}{m}$$



For electrons: f_c = 28 B GHz (B in Tesla)

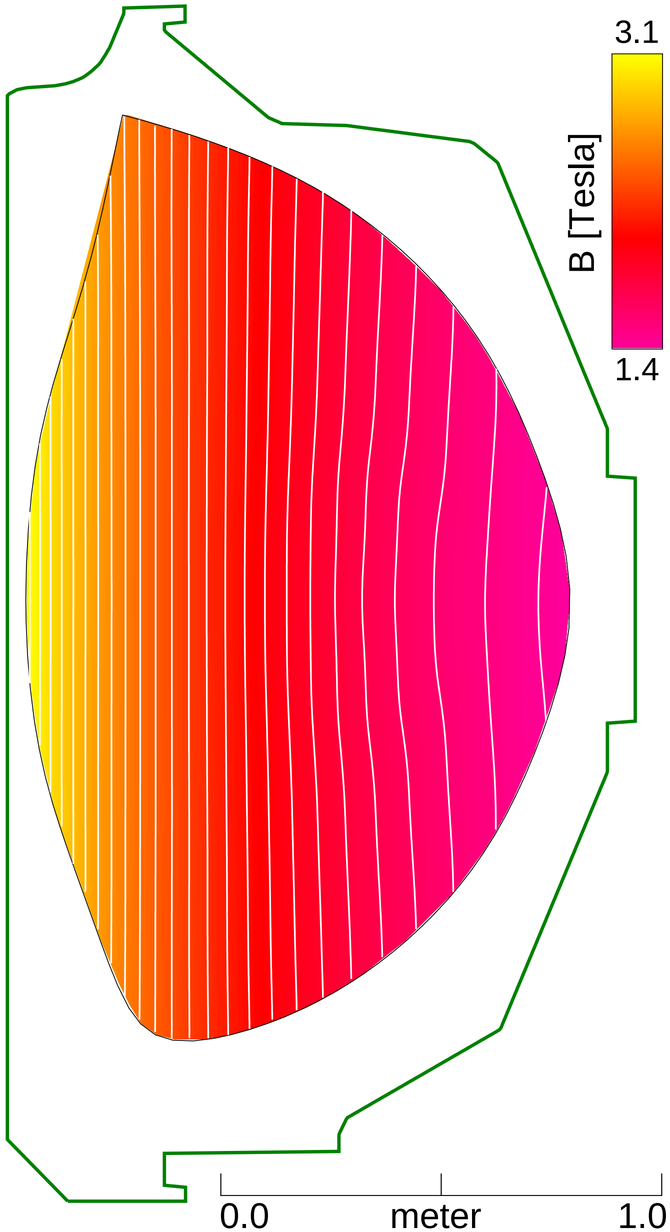
Temperature measurements of magnitized electrons electron cyclotron emission in a tokamak



- The magnetic field varies with the major radius:

$$B = \frac{R_0 B_0}{R}$$

Temperature measurements of magnitized electrons electron cyclotron emission in a tokamak



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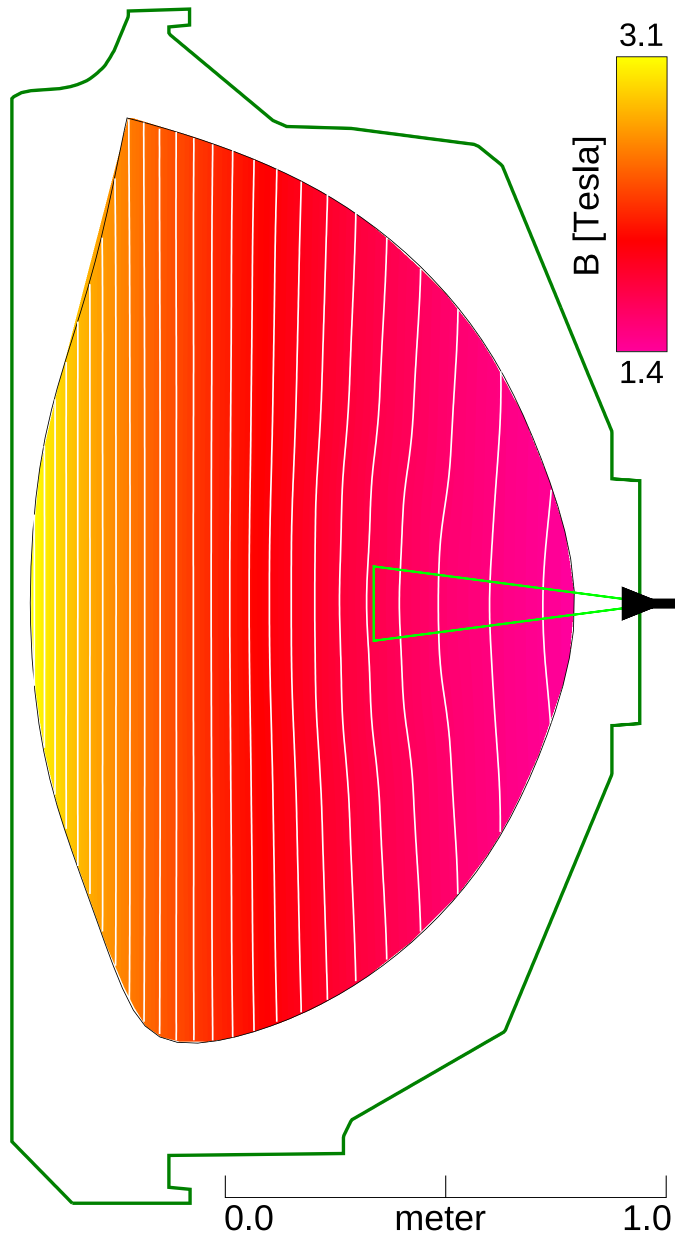
- Therefore, ECE resonances form vertical layers:

$$\omega_c = \frac{qB}{m}$$

Temperature measurements of magnitized electrons electron cyclotron emission in a tokamak



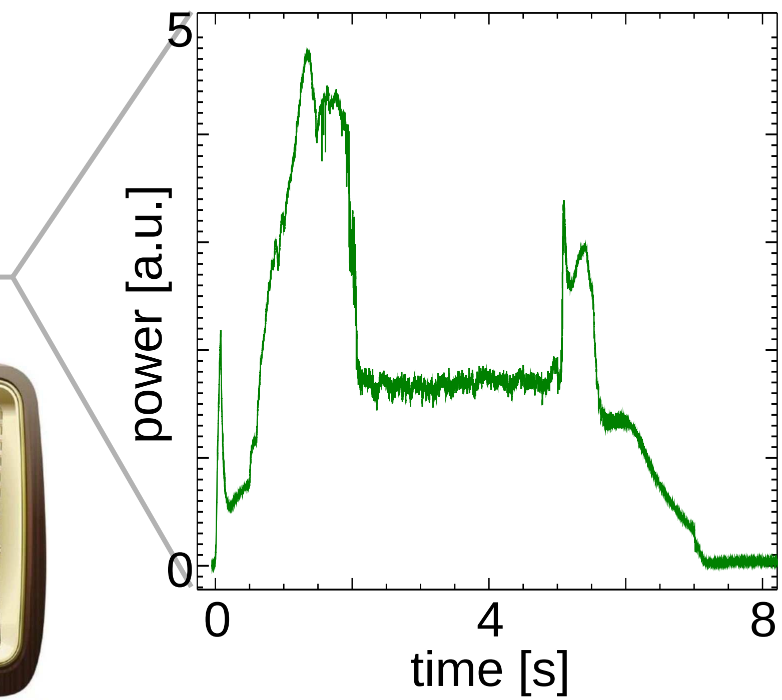
- The magnetic field varies with the major radius:

$$B = \frac{R_0 B_0}{R}$$

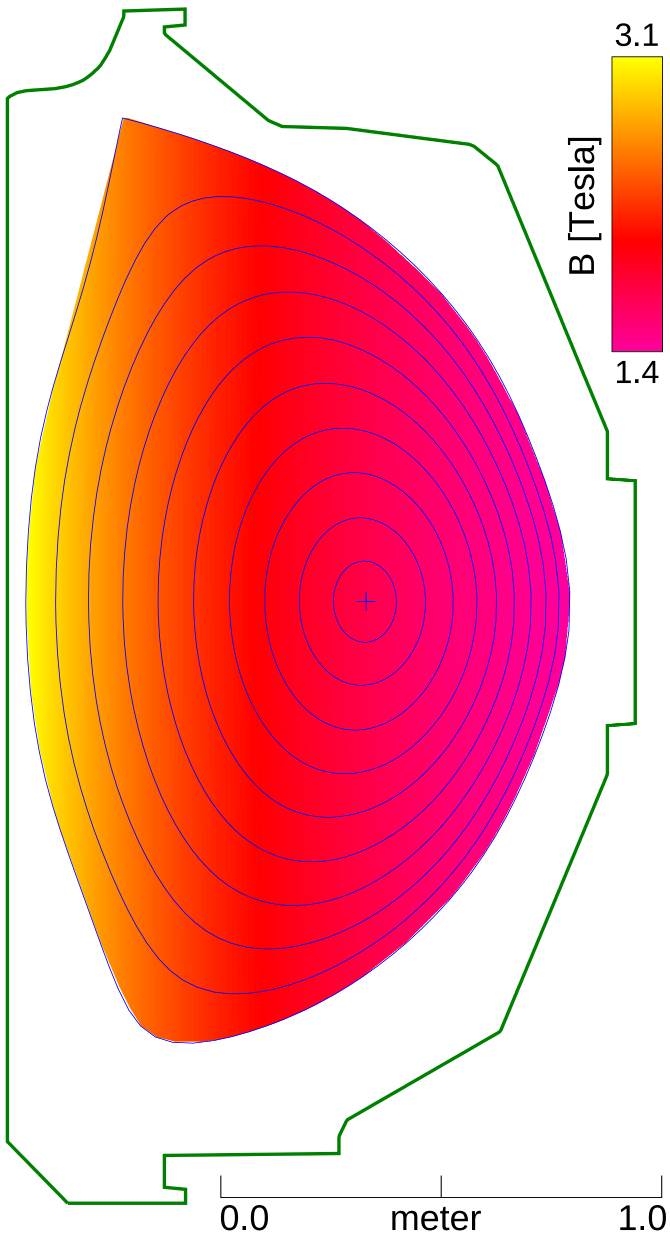
- Therefore, ECE resonances form vertical layers:

$$\omega_c = \frac{qB}{m}$$

- And local electron cyclotron emission can be measured



Temperature measurements of magnitized electrons electron cyclotron emission in a tokamak

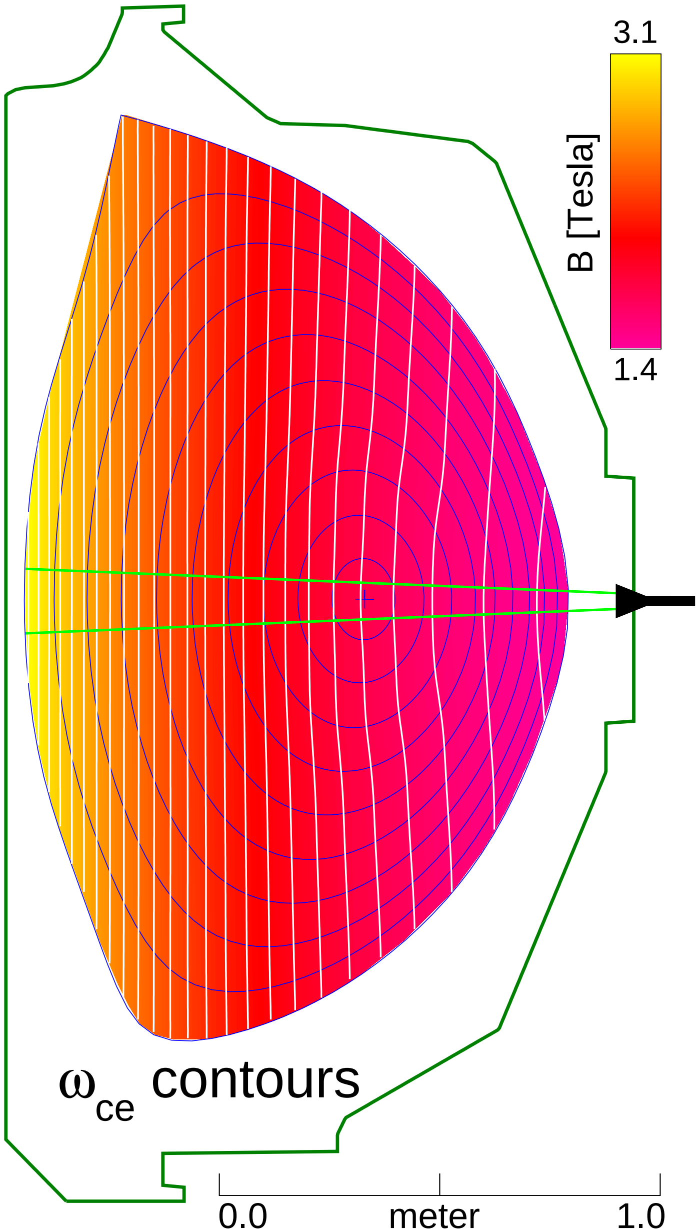


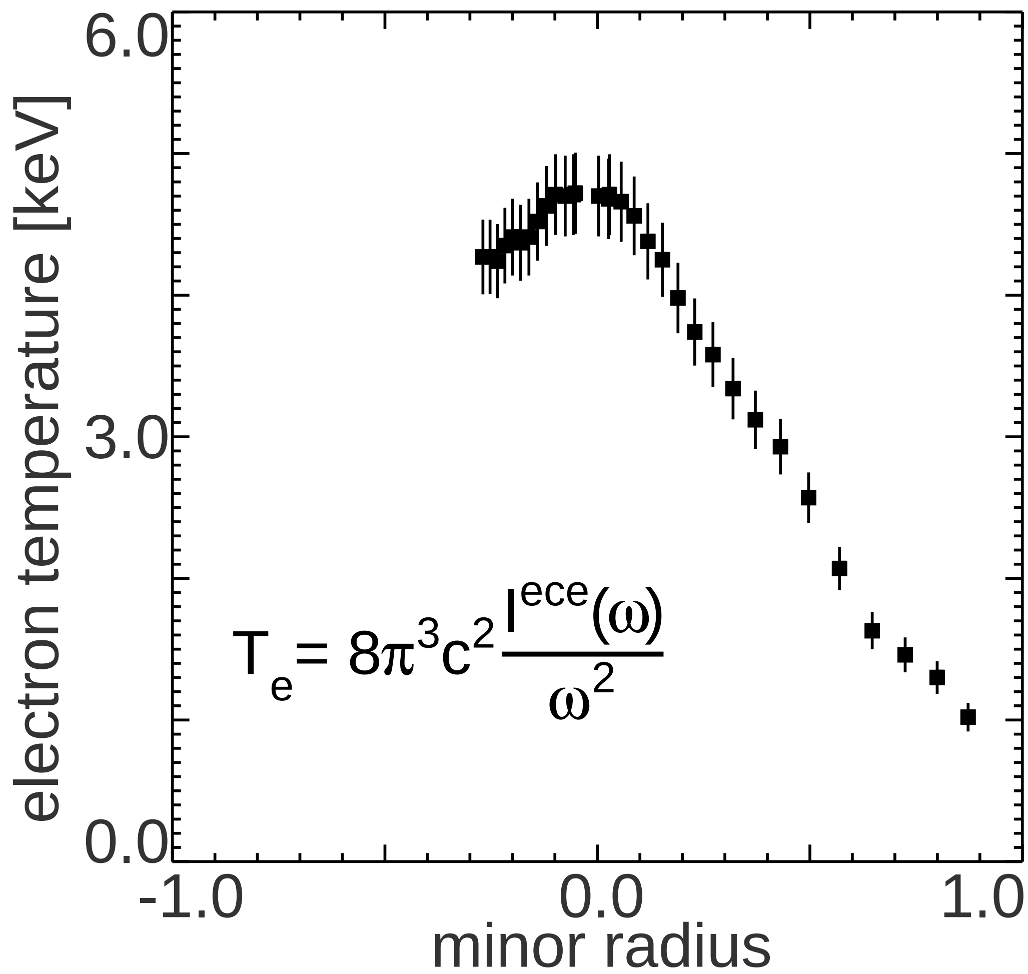
- The electron temperature is constant on magnetic flux surfaces
- When the plasma is dense enough it acts as a Black Body in thermal equilibrium
- Kirchhoff's law of thermal radiation: Emitted radiation is equal to absorbed radiation
- Therefore, the measured ECE radiation gives the local electron temperature:

$$I^{ece}(\omega) = \frac{\omega^2 T_e}{8\pi^3 c^2}$$

Rayleigh-Jeans

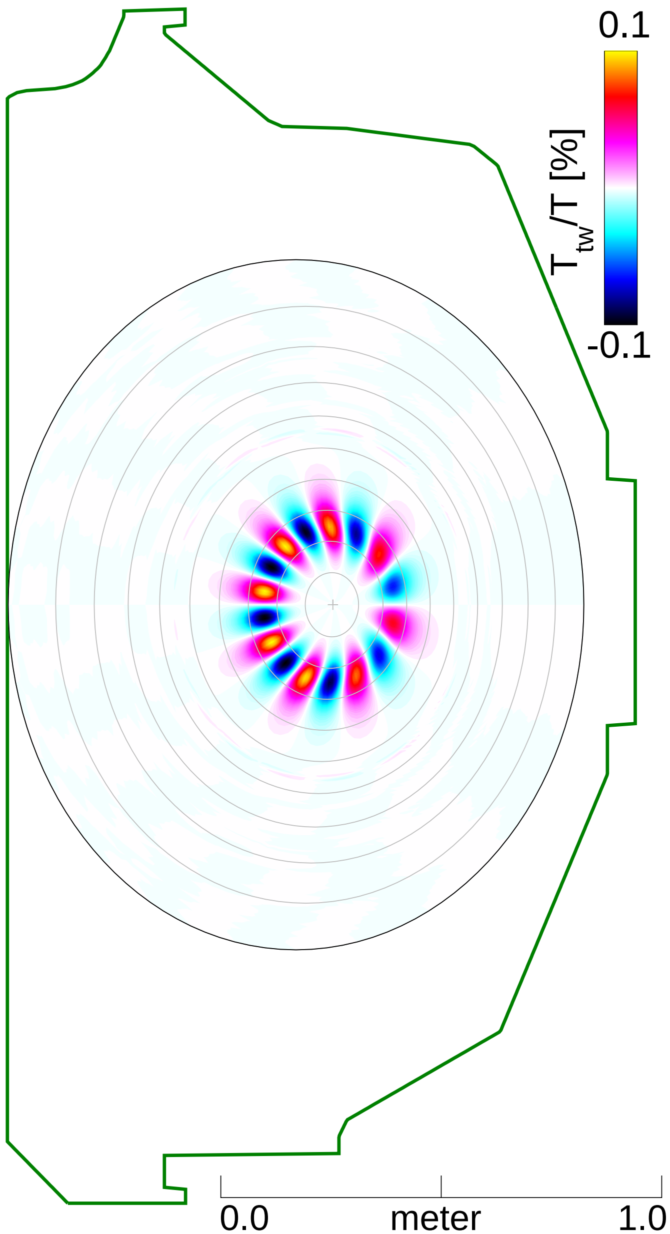
Temperature measurements of magnitized electrons electron cyclotron emission in a tokamak





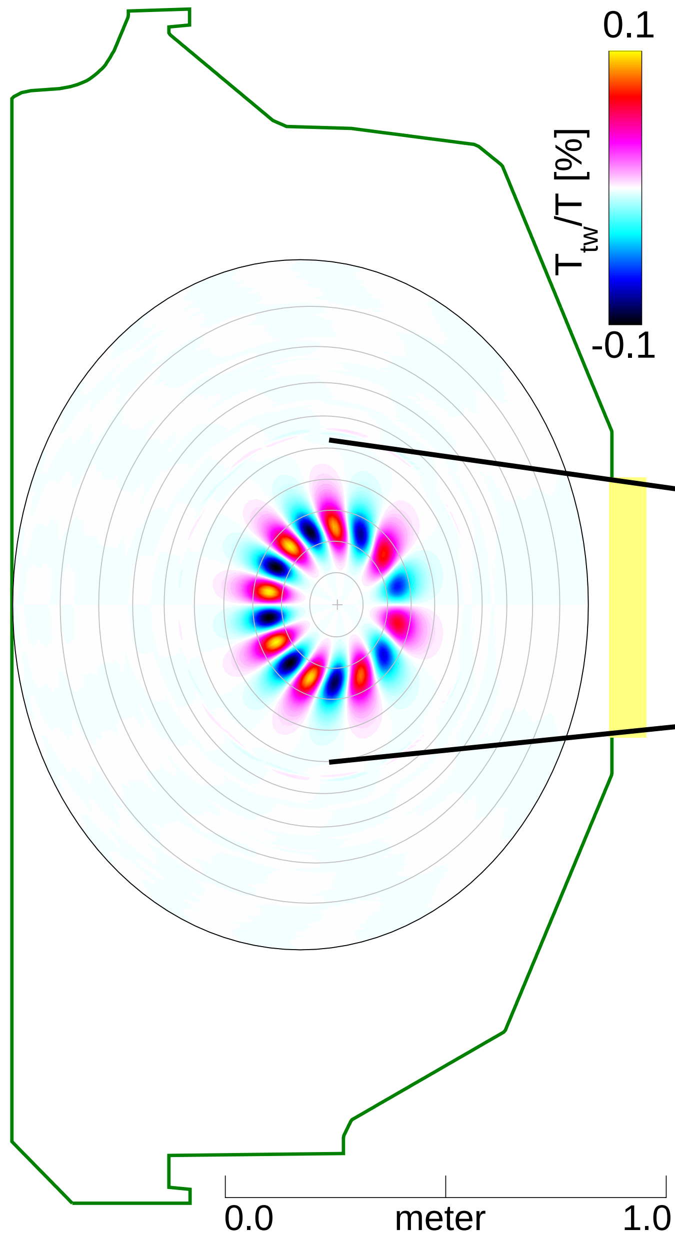
- Measurement of ECE radiation on the plasma mid-plane at different freqiencies gives the equilibrium electron temperature profile

Temperature measurements of magnitized electrons
Taking images of temperature fluctuations



- The electron temperature can deviate locally from the average temperature due to:
  - distortions of the flux surfaces
  - plasma waves
  - turbulence

Temperature measurements of magnitized electrons
Taking images of temperature fluctuations



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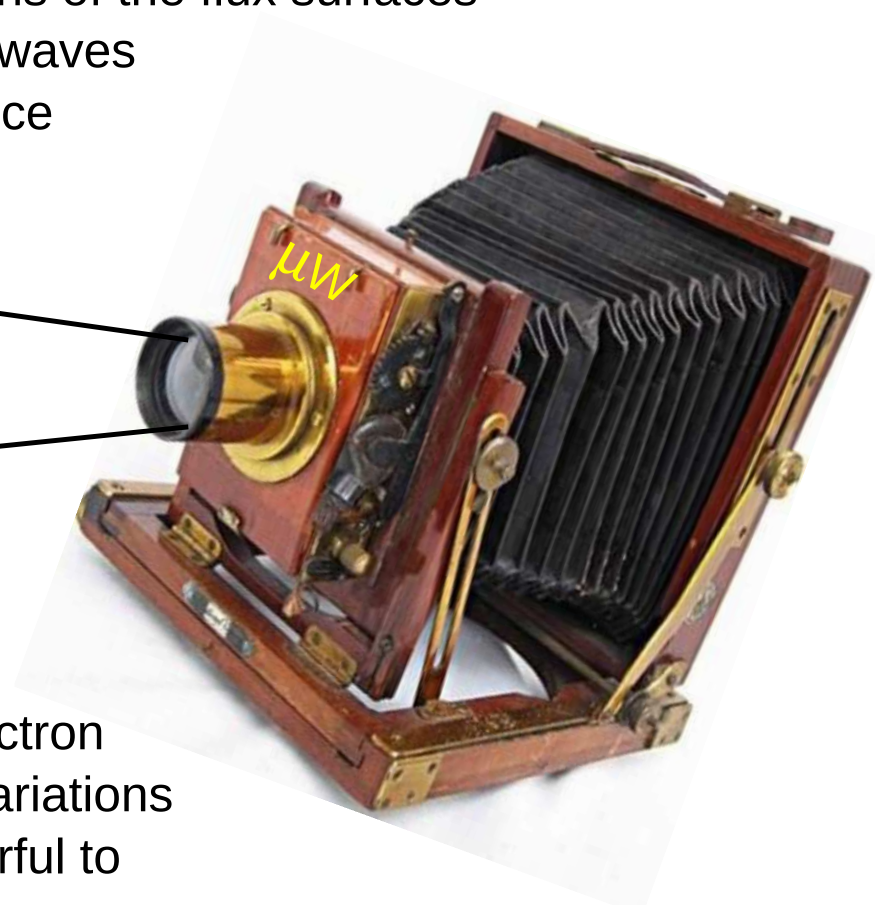
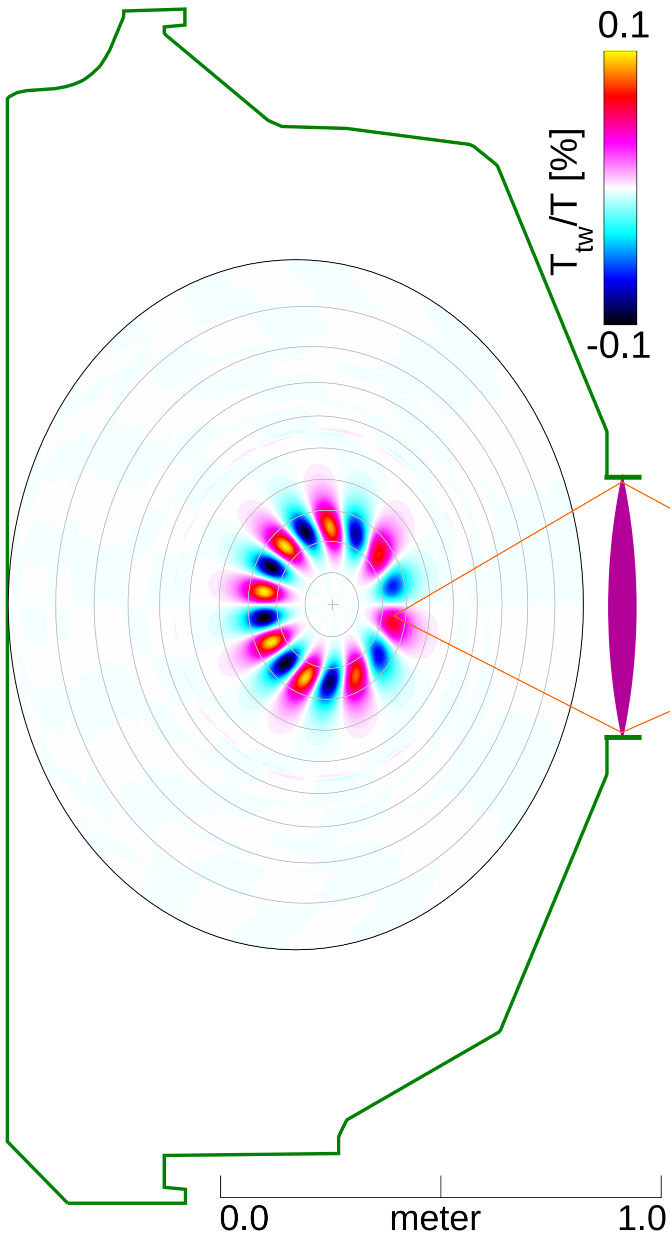
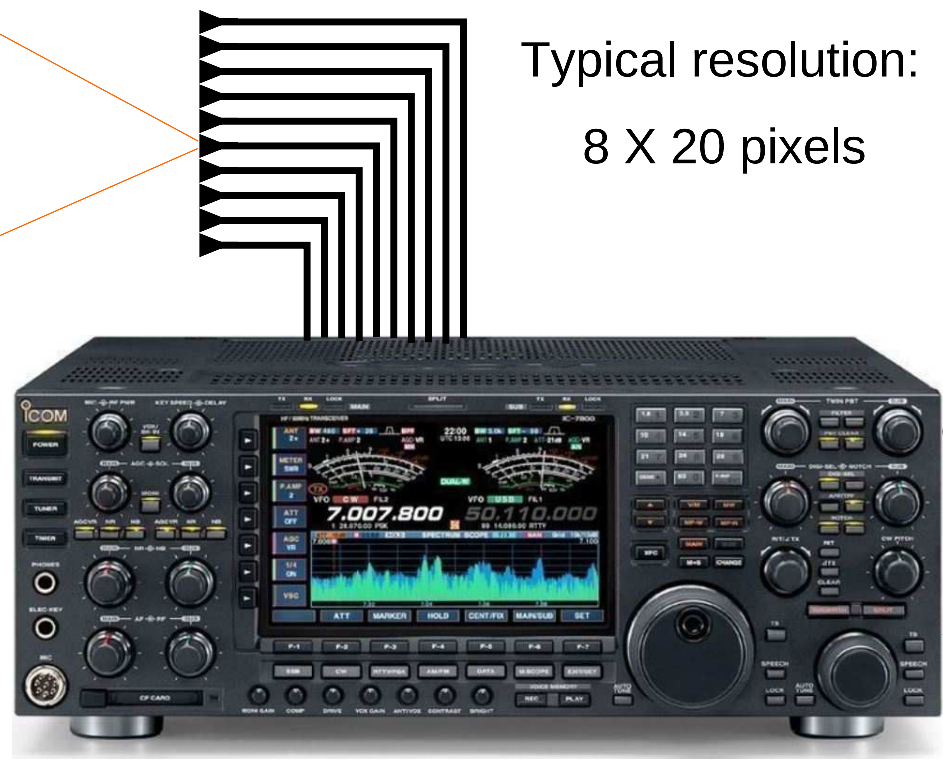


- Pictures of electron temperature variations is a very powerful to study plasmas

Temperature measurements of magnitized electrons
Taking images of temperature fluctuations

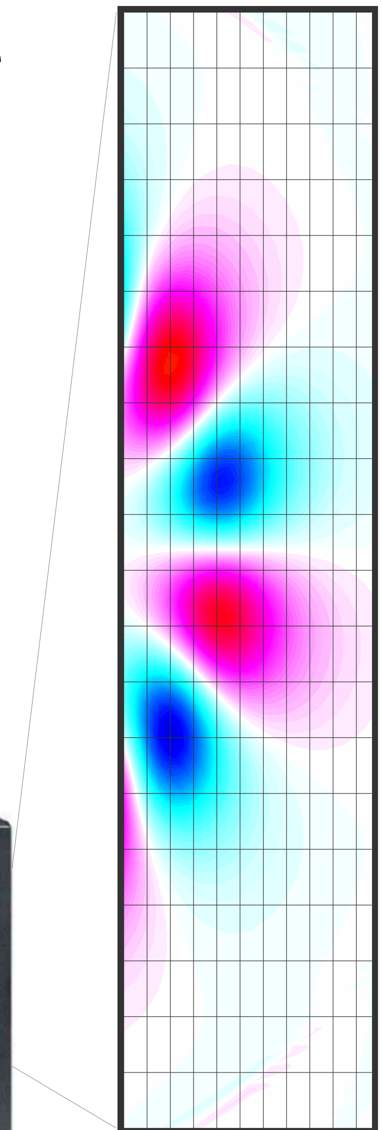


- An array of antennas with receivers can be used to image a vertical plane (constant ω_c)
- A lens is used to image a part of the plasma on an array of antennas

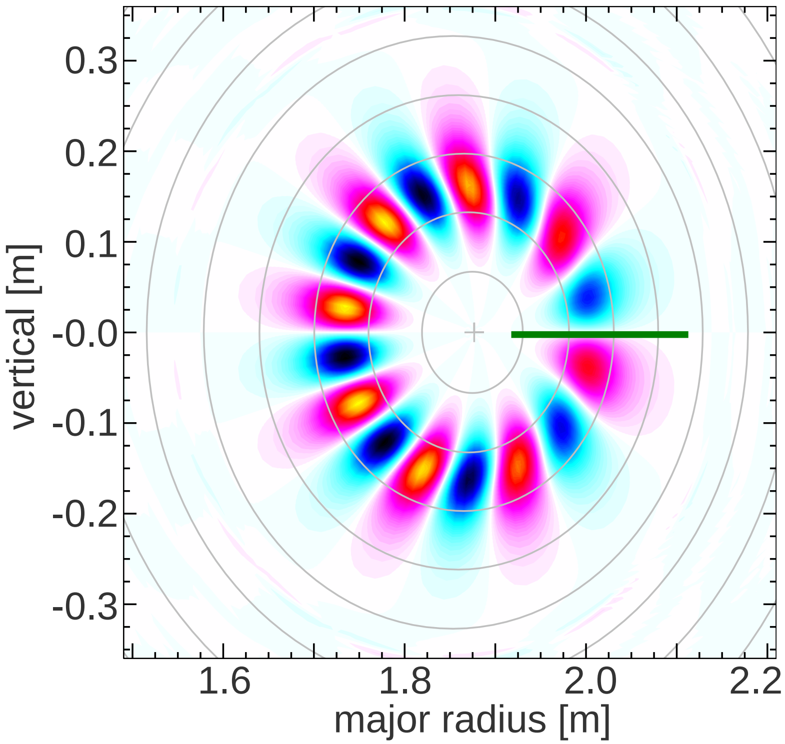


Typical resolution:

8 X 20 pixels

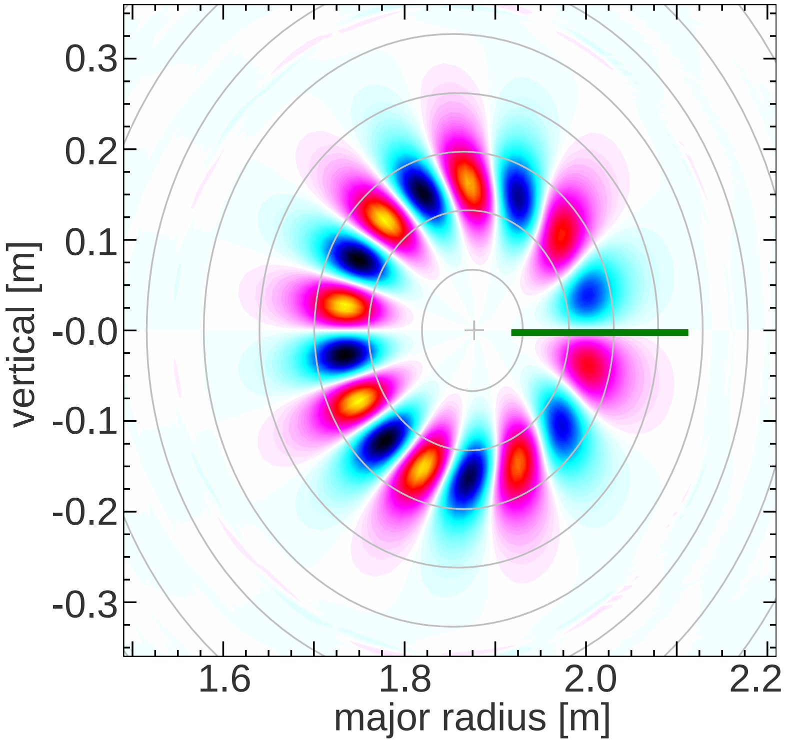


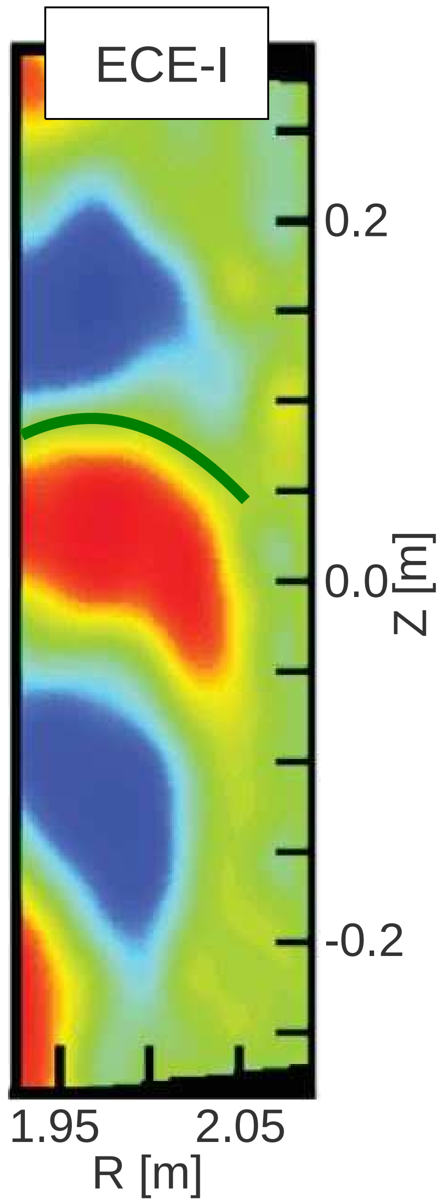
Temperature measurements of magnitized electrons ECE images help to direct theory development



- Ideal MHD theory predicts that the radial wave front of Alfvén waves are constant

Temperature measurements of magnitized electrons ECE images help to direct theory development

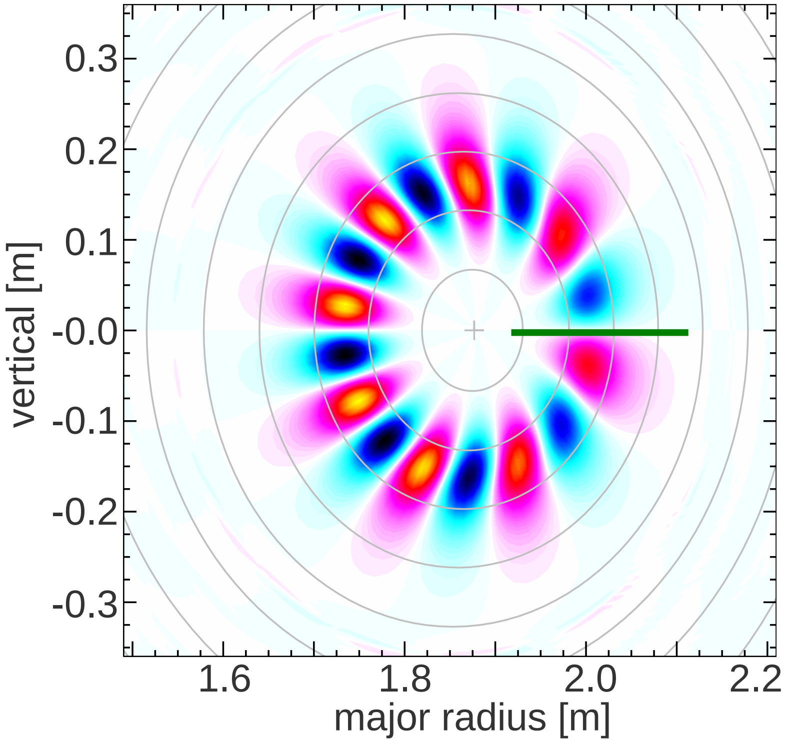


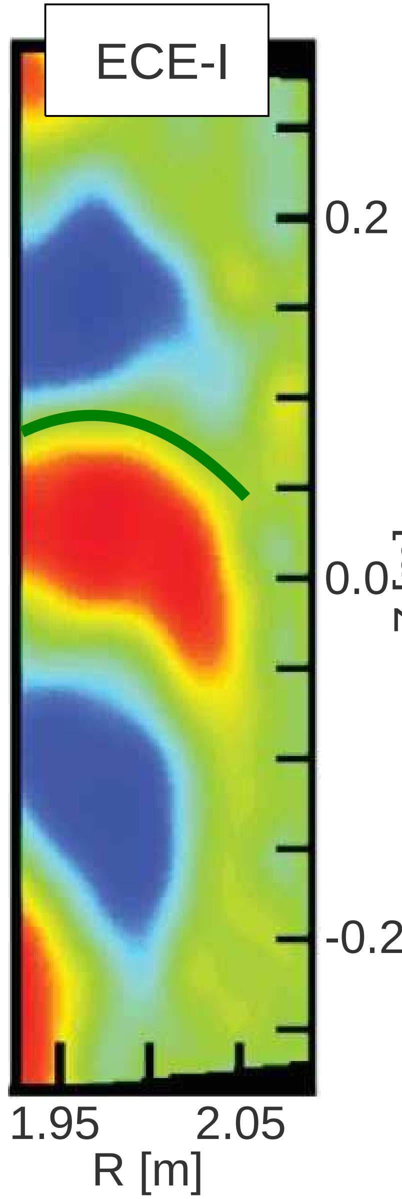


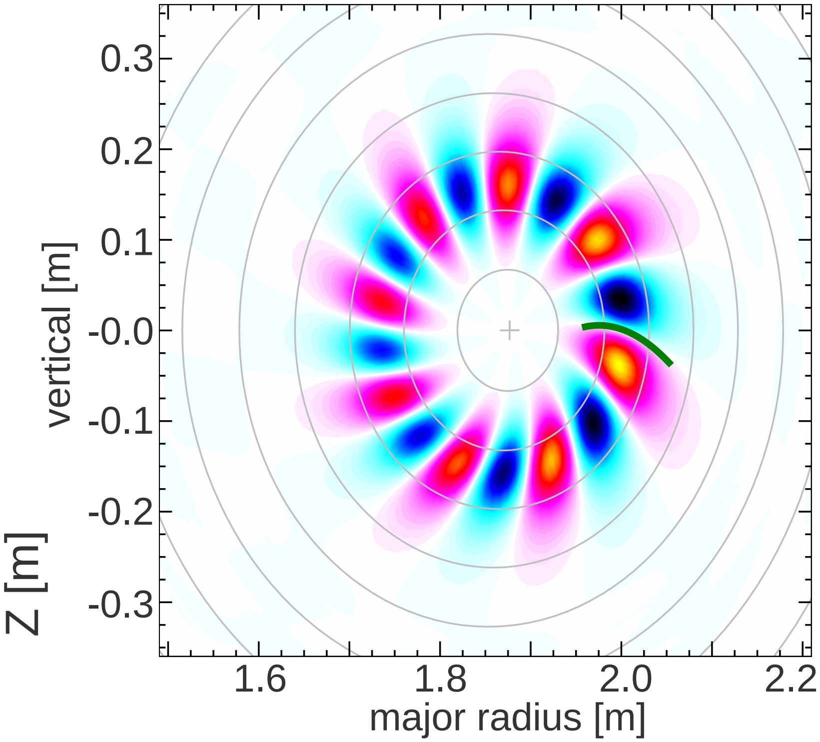
- Ideal MHD theory predicts that the radial wave front of Alfvén waves are constant

- ECE images show that the wave fronts are radially curved

Temperature measurements of magnitized electrons
ECE images help to direct theory development







- Ideal MHD theory predicts that the radial wave front of Alfvén waves are constant
- ECE images show that the wave fronts are radially curved

- Wave front curvature indicates effects beyond ideal MHD
- Ad-hoc radial phase variation indicates effects on energy transport and magnetic fields

Probing plasmas with microwave beams
Electromagnetic waves in plasmas

- From Maxwell's equations we can derive a wave equation:

$$\nabla \times (\nabla \times \mathbf{E}) + \partial_t(\mu_0 \mathbf{j} + \varepsilon_0 \mu_0 \partial_t \mathbf{E}) = 0$$

- The plasma response to the waves is given by current density:

$$\mathbf{j} = q\, n\, \mathbf{v} = \sigma \cdot \mathbf{E}$$

- Because electrons are much more mobile than ions we can consider the movement of the electrons to the plasma response to the waves and evaluate the dielectric tensor:

$$\varepsilon = (1 + \frac{i}{\omega \varepsilon_0} \sigma)$$

- Using the Lorentz equation we can derive the Appleton-Hartree equation which gives the dielectric tensor in terms of the plasma and cyclotron frequencies:

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}} \qquad \omega_{ce} = \frac{eB}{m_e}$$

Probing plasmas with microwave beams
Electromagnetic waves in plasmas

- The wave equation in Fourier space reads:

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) - \frac{\omega^2}{c^2} \varepsilon \mathbf{E} = 0$$

- and the dielectric tensor:

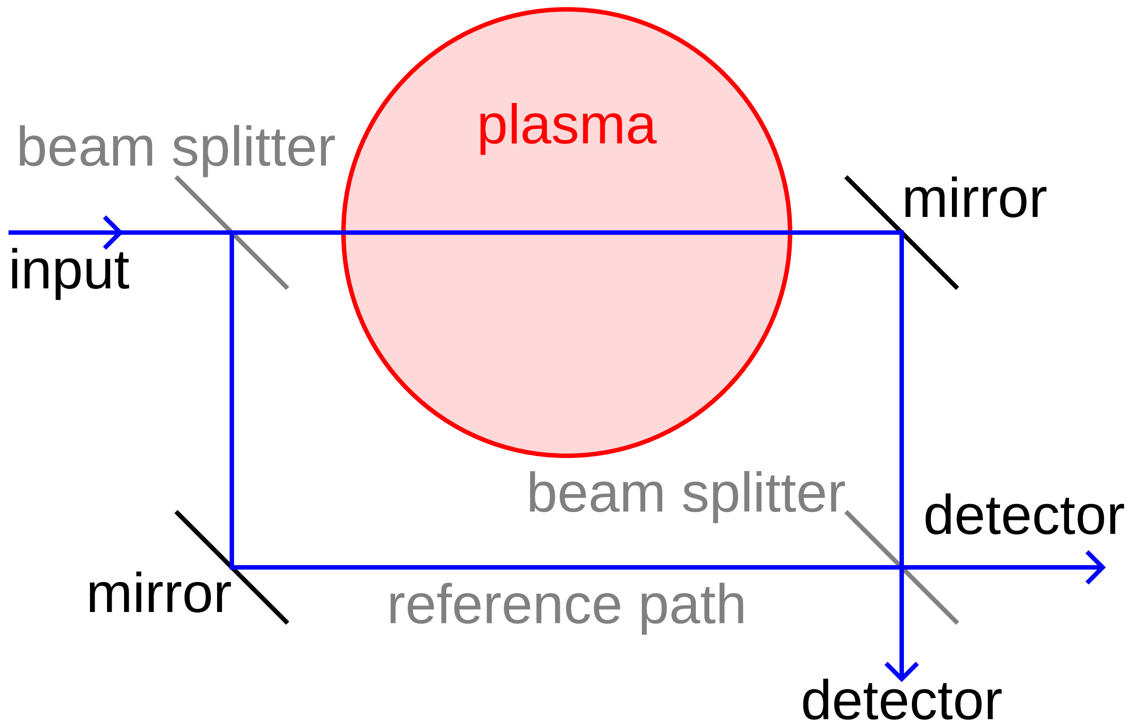
$$\varepsilon(\mathbf{x}) = (1 + \frac{i}{\omega \varepsilon_0} \sigma(\mathbf{x}))$$

determines the properties of the wave:

$\varepsilon > 0$ normal wave

Probing plasmas with microwave beams
Interferometry for line-integrated electron densities

- The phase difference between the plasma path and the reference path is proportional to the line averaged density



- phase in plasma path:

$$\varphi_{pls} = \int \sqrt{\varepsilon}\,\frac{\omega}{c}\,dl$$

- phase in reference path:

$$\varphi_{ref} = \int \frac{\omega}{c}\,dl$$

- The phase difference gives the line integrated electron density:

$$\Delta\varphi = \int \frac{\omega}{c}(\sqrt{\varepsilon} - 1)\,dl \approx \frac{\omega}{2cn_{crit}} \int n_e\,dl$$

$$\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega^2}$$

Condition: $\omega_{pe} << \omega$

$$\omega_{pe}^2 = \frac{n_e e^2}{\varepsilon_0 m_e} \qquad n_{crit} = \frac{\varepsilon_0 m_e \omega^2}{e^2}$$

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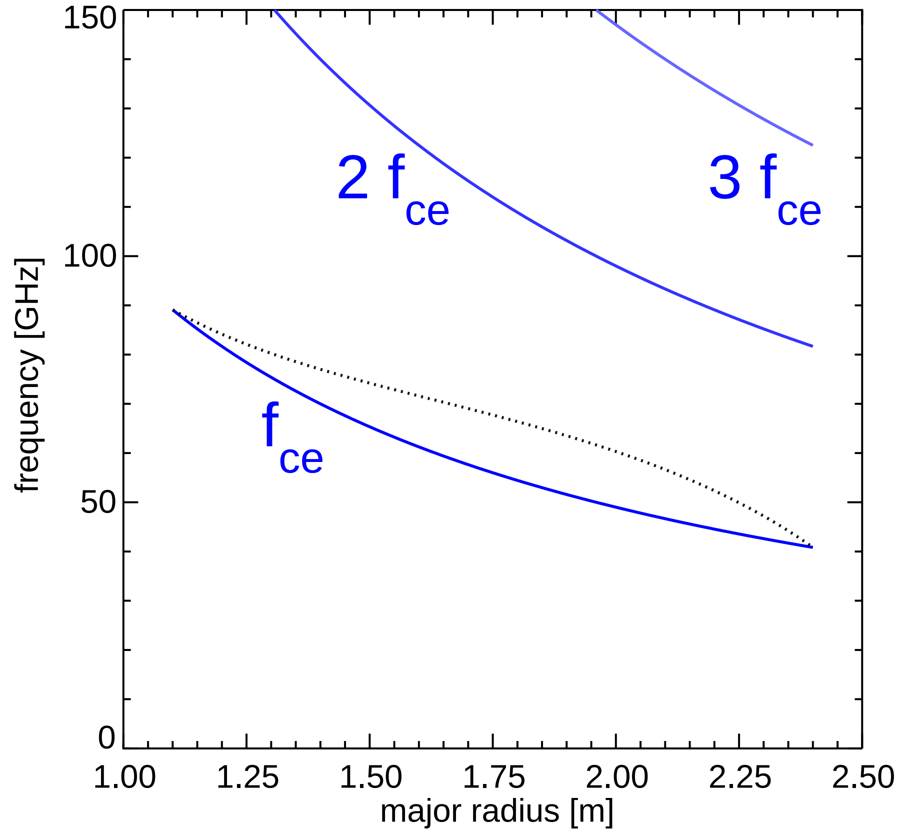
determines the properties of the wave:

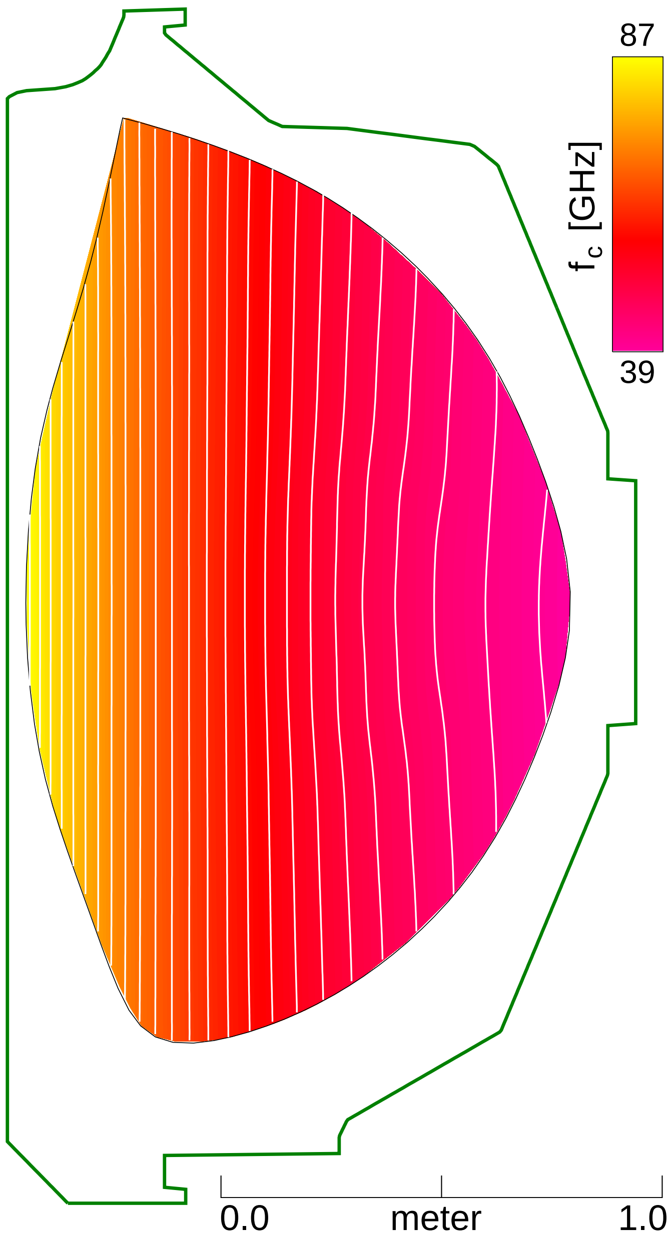
$\varepsilon > 0$ normal wave

$\varepsilon \to \infty$ resonance



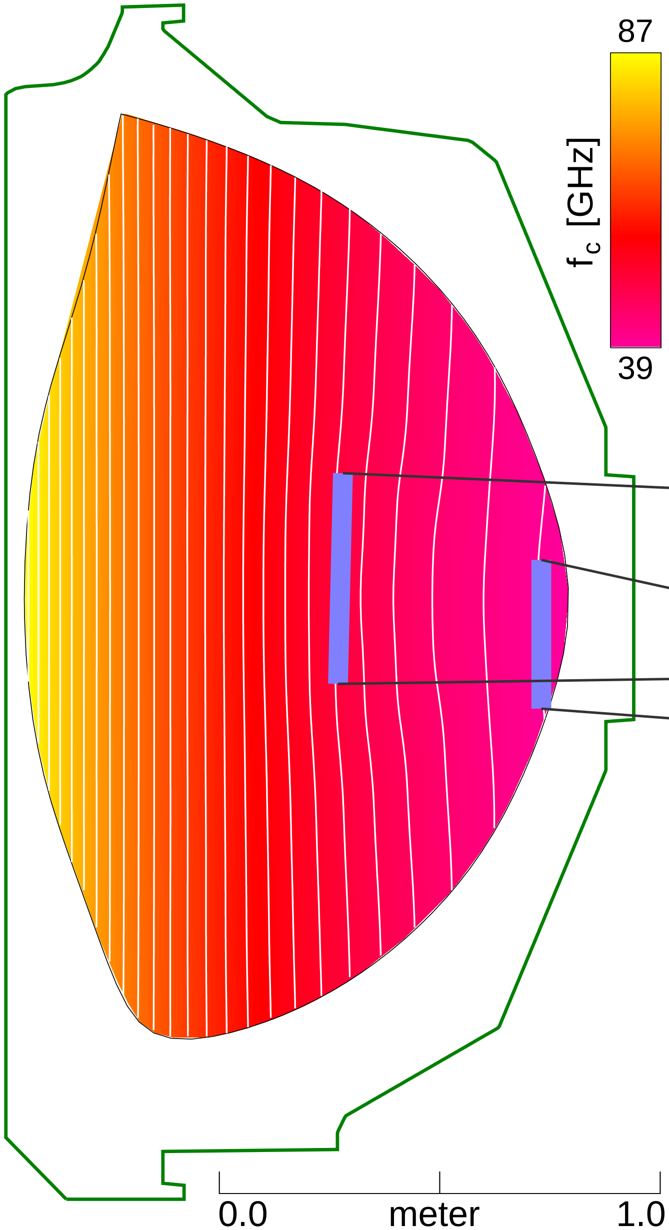
$$\omega_{ce} = \frac{eB}{m_e} \qquad \omega_{pe} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

Probing plasmas with microwave beams
Resonances are great for plasma heating



- Microwave power is adsorbed at resonances
- This is not useful for diagnostic puroposes

Probing plasmas with microwave beams
Resonances are great for plasma heating



- Microwave power is adsorbed at resonances
- This is not useful for diagnostic puroposes
- It is great for heating and driving current locally in fusion plasmas

Probing plasmas with microwave beams
Electromagnetic waves in plasmas

- The wave equation in Fourier space reads:

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) - \frac{\omega^2}{c^2} \varepsilon \mathbf{E} = 0$$

- and the dielectric tensor:

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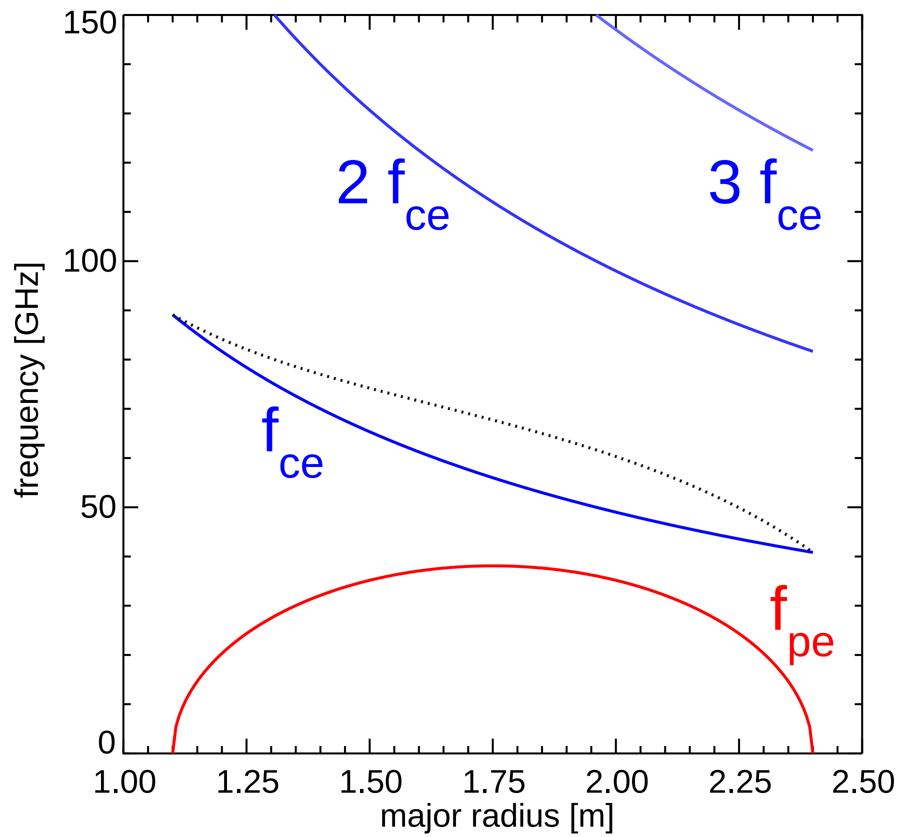
determines the properties of the wave:

$\varepsilon > 0$	normal wave
$\varepsilon \to \infty$	resonance
$\varepsilon = 0$	cut-off

O-mode cut-off: $\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega^2}$



$$\omega_{ce} = \frac{eB}{m_e} \qquad \omega_{pe} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

Probing plasmas with microwave beams
Electromagnetic waves in plasmas

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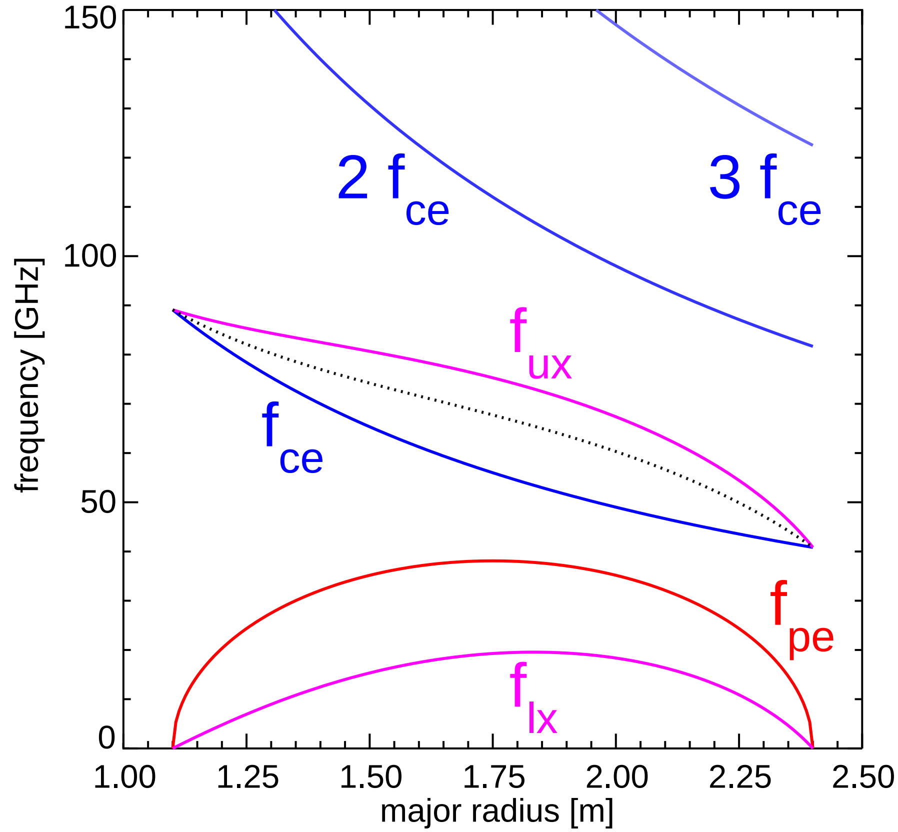

O-mode cut-off: $\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega^2}$

X-mode cut-off: $\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega^2} \frac{\omega - \omega_{pe}^2}{\omega - \omega_{pe}^2 - \omega_{ce}^2}$

$$\omega_{ce} = \frac{eB}{m_e} \qquad \omega_{pe} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

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determines the properties of the wave:

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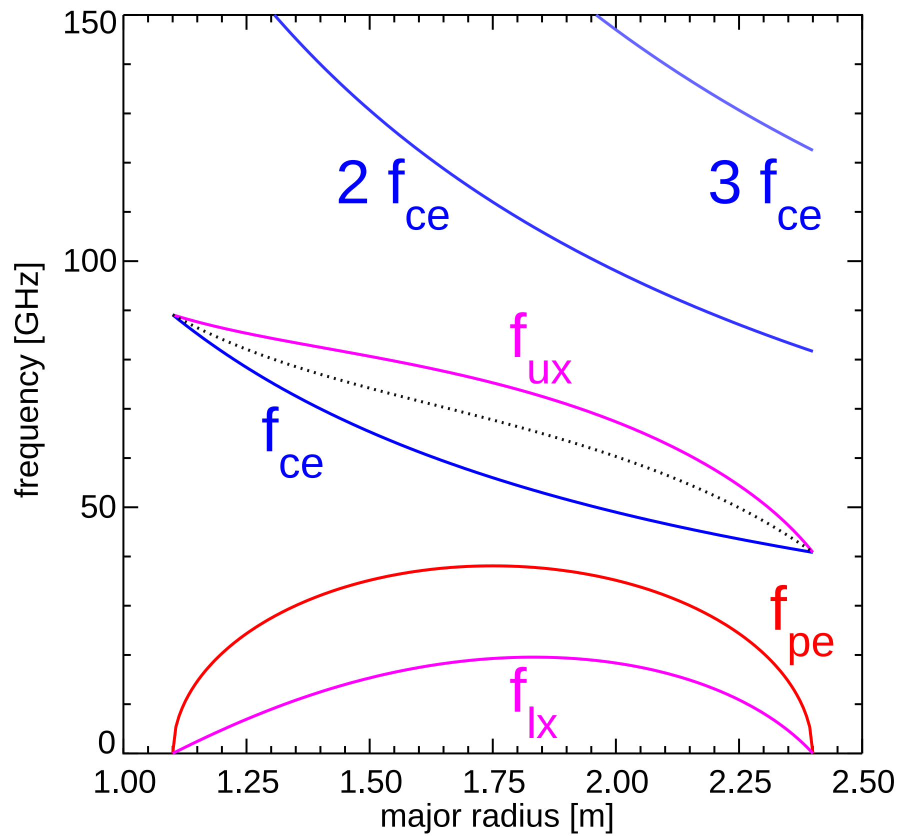
$\varepsilon \rightarrow \infty$	resonance
$\varepsilon = 0$	cut-off
$\varepsilon < 0$	evanescent wave



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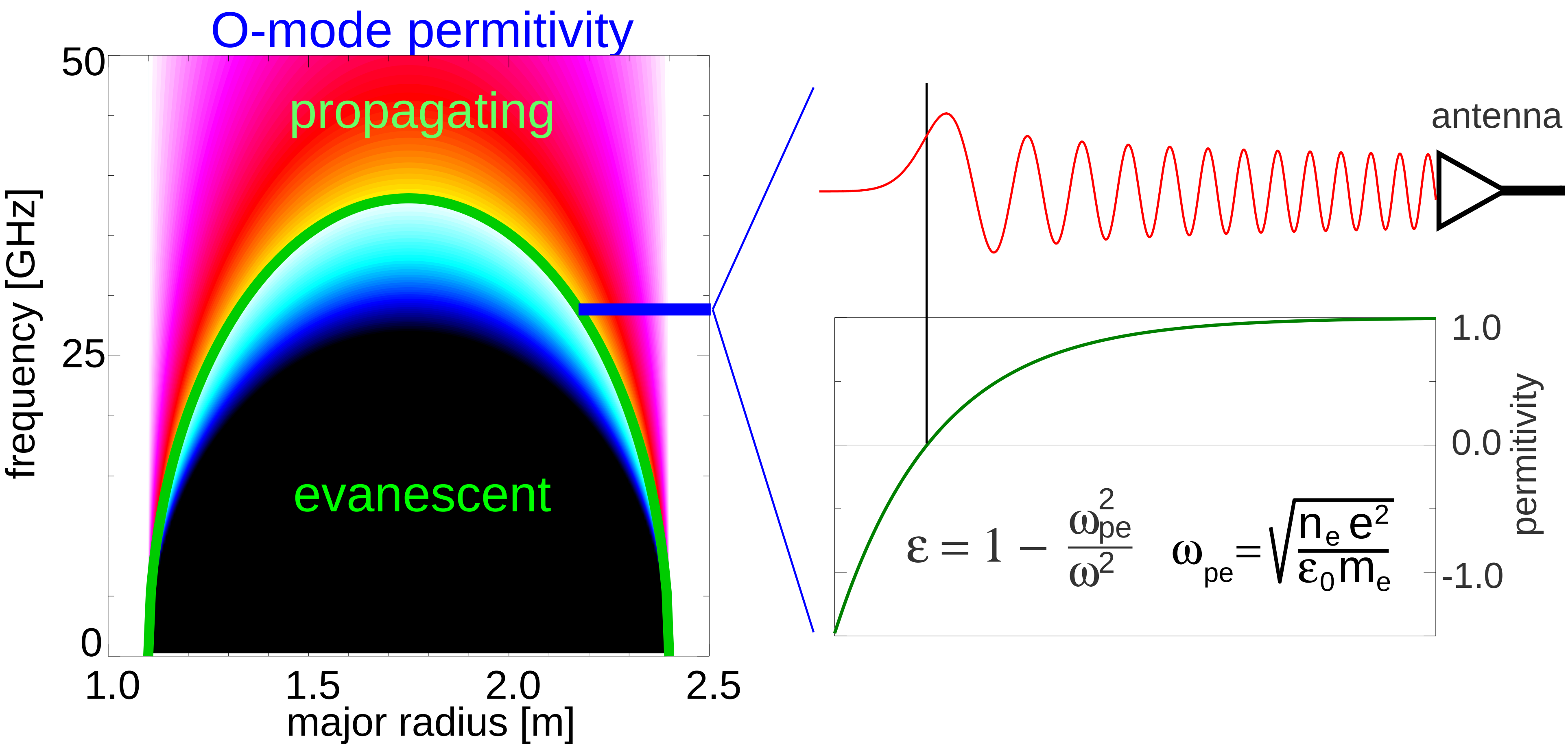
X-mode cut-off: $\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega^2} \frac{\omega - \omega_{pe}^2}{\omega - \omega_{pe}^2 - \omega_{ce}^2}$

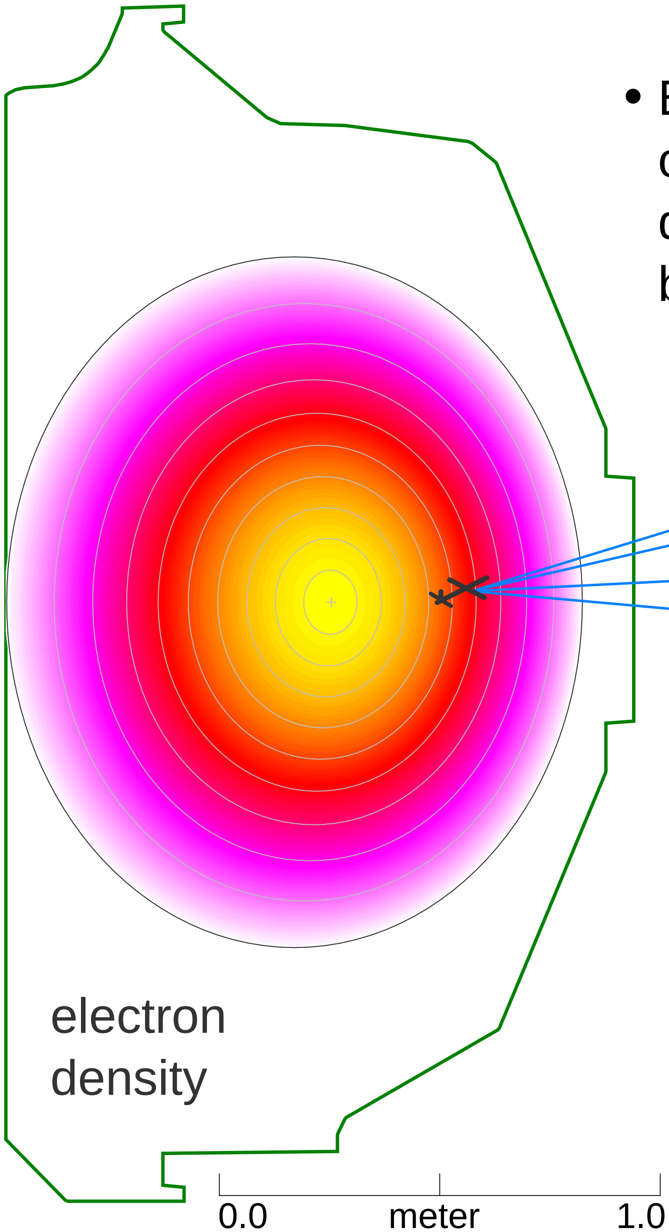
$$\omega_{ce} = \frac{eB}{m_e} \qquad \omega_{pe} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

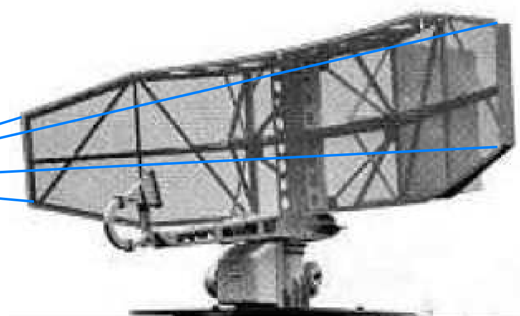
Local electron density measurements
Waves are reflected from cut-off layers



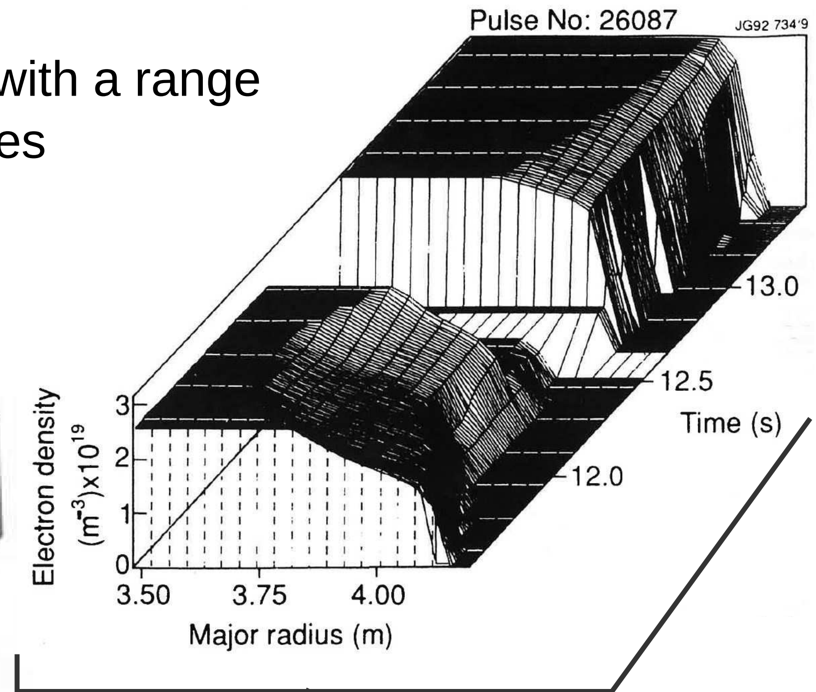
- Cut-off layers can be probed from outside the plasma
- The wave frequency determines the reflection point in the plasma
- The cut-off location in the plasma is obtained from measuring the wave's round trip time
- A great diagnostic to obtain density profiles

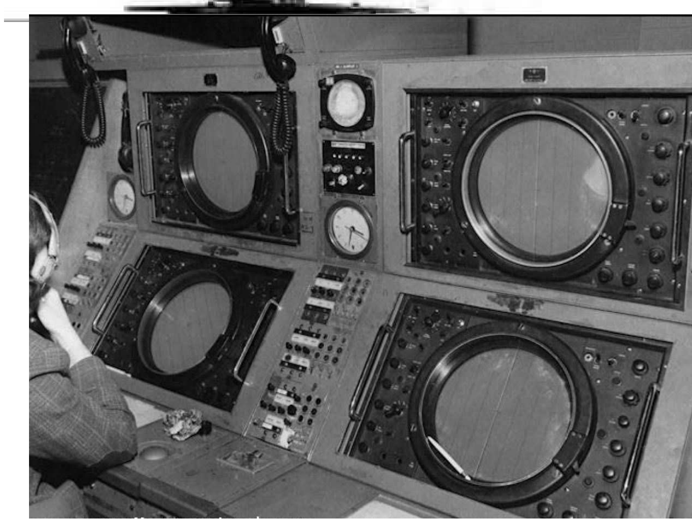
Local electron density measurements
Measurement of time resolved density profiles





- By probing the plasma with a range of microwave frequencies density profiles can be measured





- Electron density profile measurements are crucial to understand the behavior of plasmas in tokamaks

Local electron density measurements
Fluctuations have distorting effects on reflections

perfect reflection
no turbulence

distorted reflection
due to turbulence

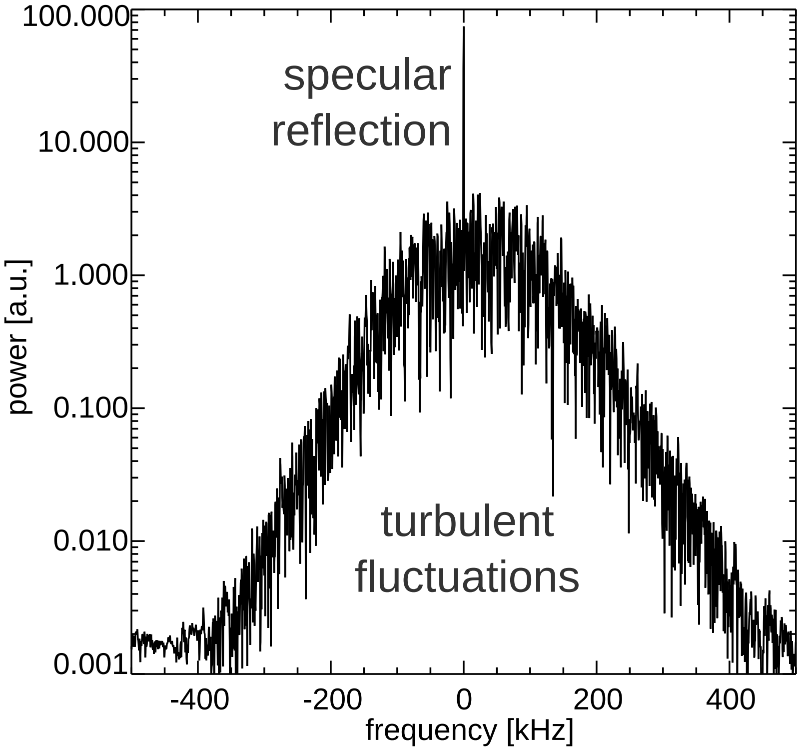
- Density fluctuations affect the reflected signal when both the wave vector:

$$\mathbf{k}_{\text{signal}} = \mathbf{k}_{\text{in}} + \mathbf{k}_{\text{turbulence}}$$

and the the frequency:

$$\omega_{\text{signal}} = \omega_{\text{in}} + \omega_{\text{turbulence}}$$

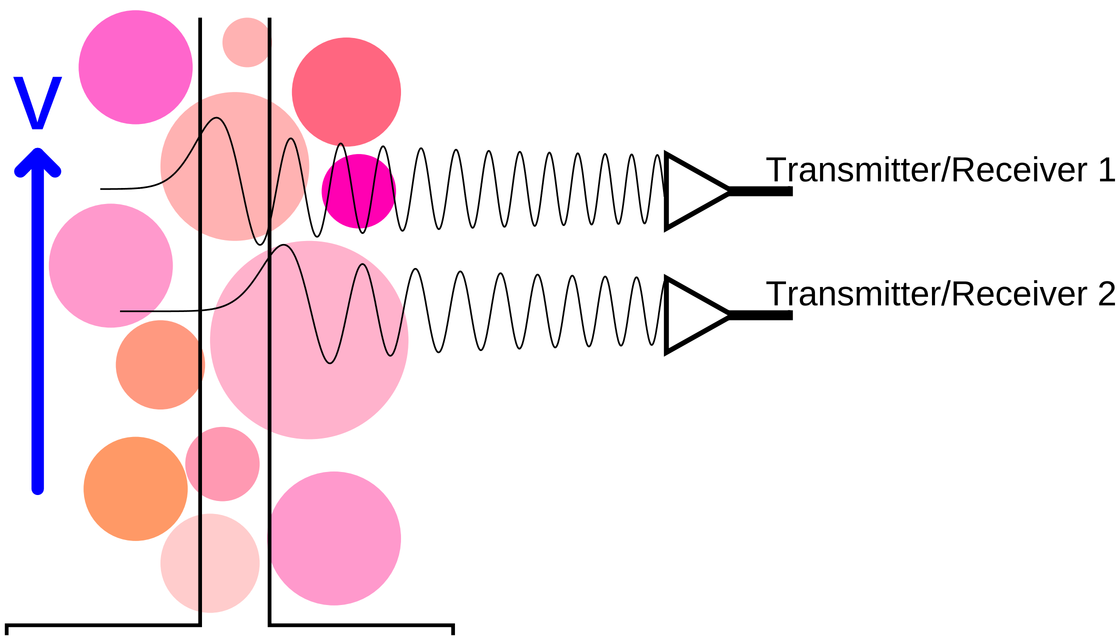
match

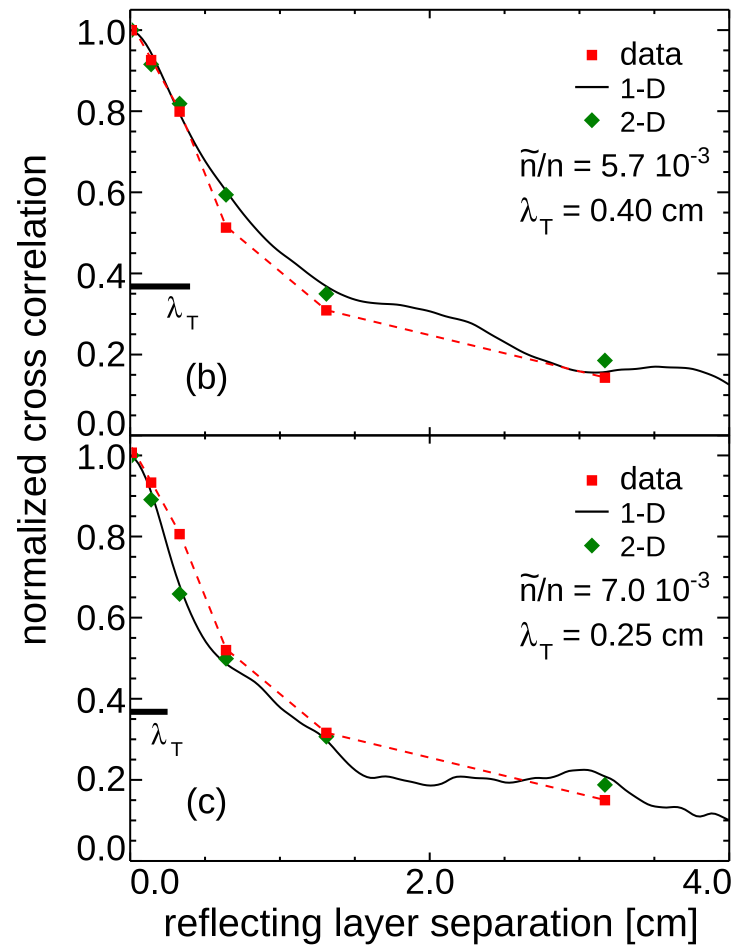


Local electron density measurements
Fluctuations carry information on plasma turbulence

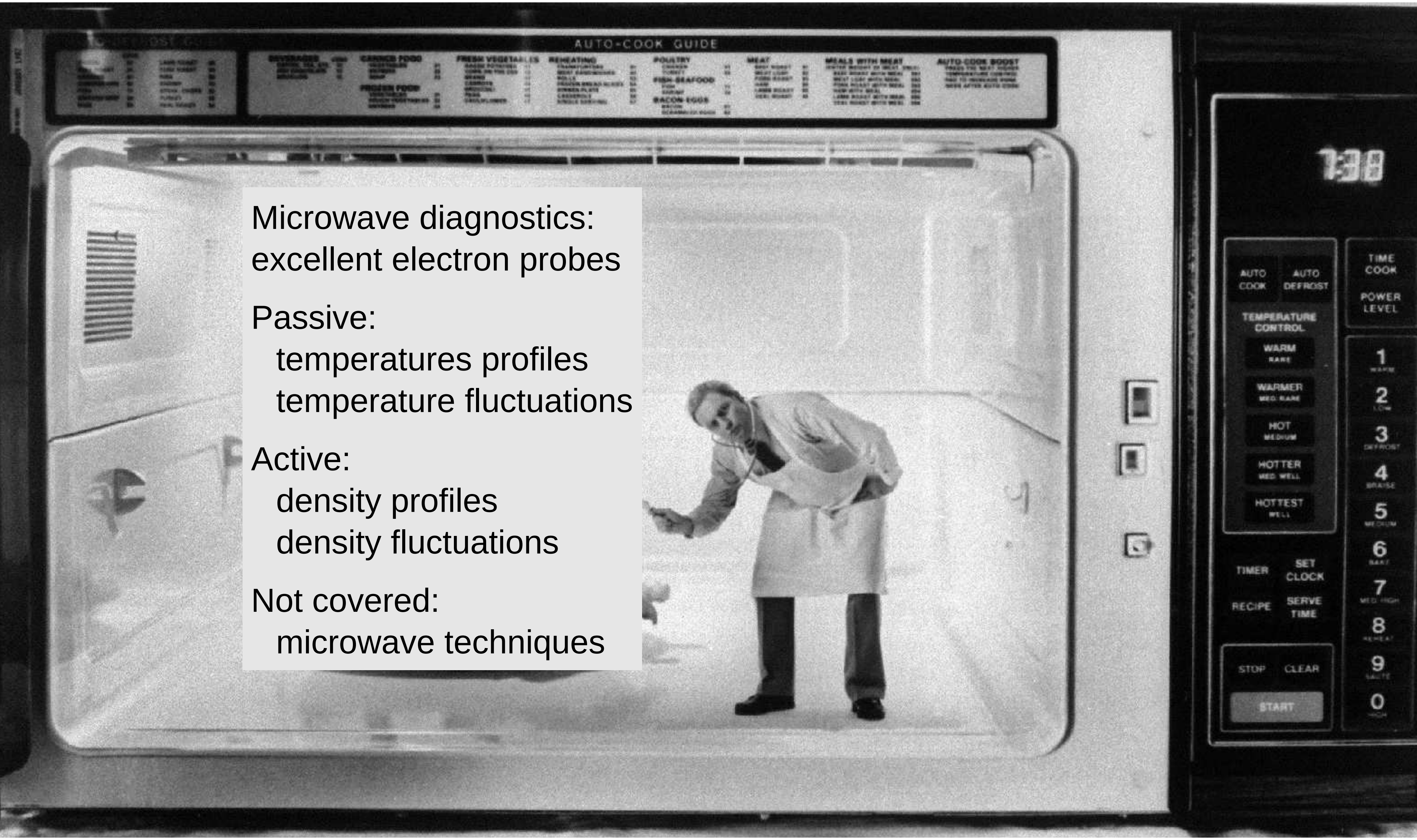
- Plasma turbulence creates random density fluctuations propagating in time and space that appears as "noise" in the signals of reflectometers

- Correlation methods are used to extract statistical properties of the plasma turbulence that can be compared to predictions from theory





Microwave Diagnostics for Plasmas



Microwave diagnostics:
excellent electron probes

Passive:
- temperatures profiles
- temperature fluctuations

Active:
- density profiles
- density fluctuations

Not covered:
- microwave techniques