Turbulence and Magneto-genesis

Steve Cowley, Archie Bott

Princeton Plasma Physics Laboratory

A Schekochihin, E. Quataert, M Kunz, N. Loureiro

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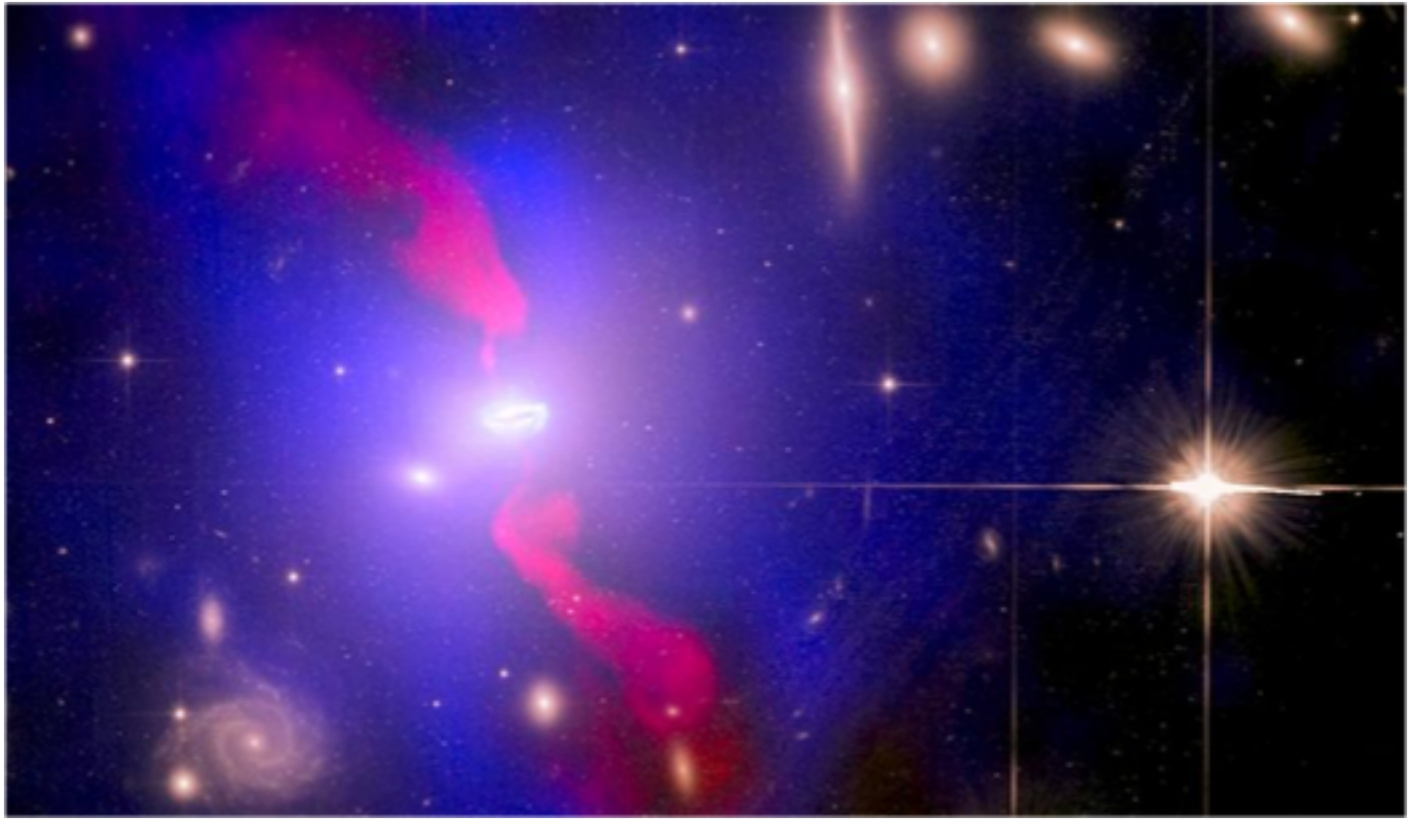
How do we describe plasmas in the universe?

When was the universe magnetized?

What does the field look like?

Is it ever laminar?

Diffuse Plasmas in Universe



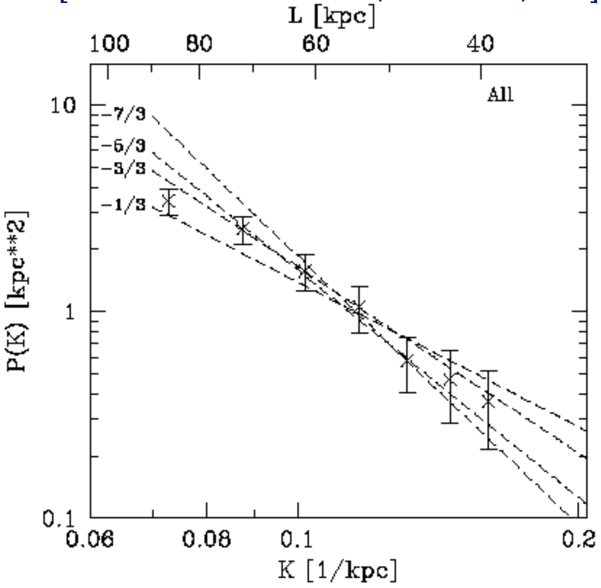
Hydra Galaxy Cluster

D Stork

Cluster MHD Turbulence

TURBULENCE
Coma cluster

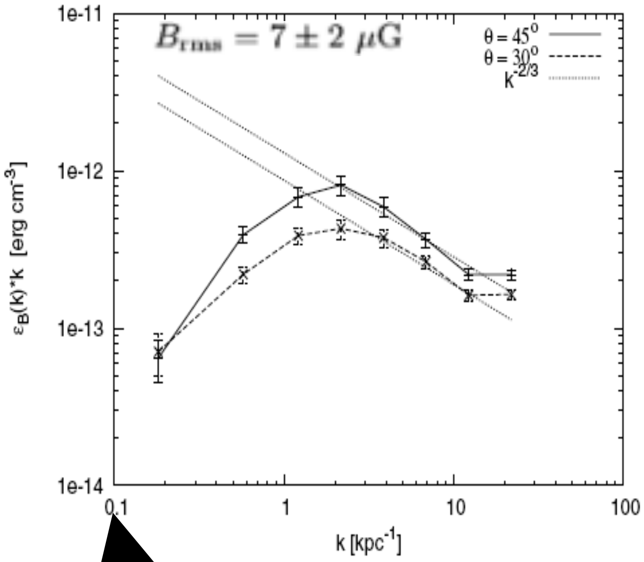
[Schuecker *et al.* 2004, *A&A* **426**, 387]



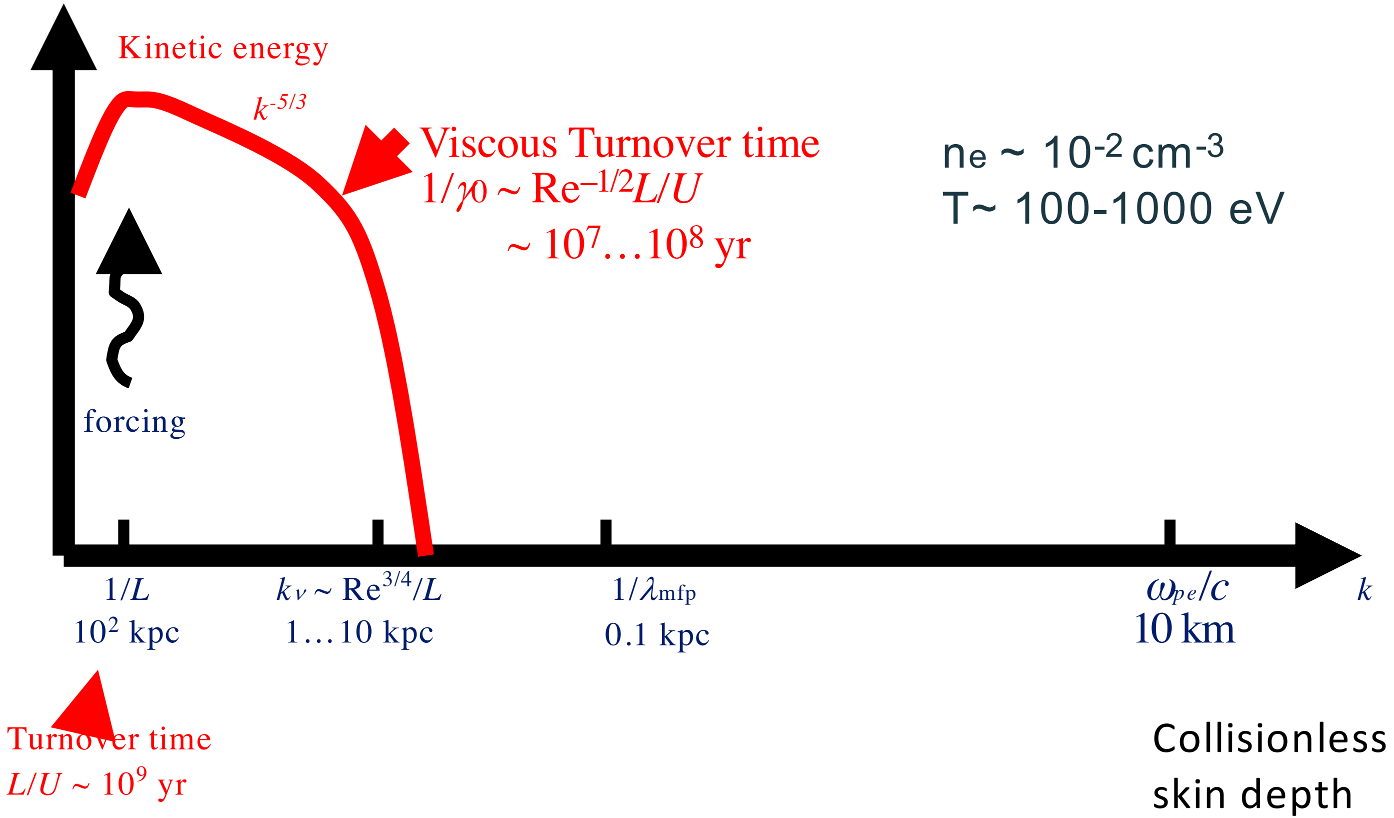
•Magnetic Reynolds #, Rm ~ 10^{29}.

MAGNETIC FIELDS
Hydra A Cluster

[Vogt & Enßlin 2005, *A&A* **434**, 67]



Turbulence scale is around here



[Schekochihin *et al.*, *ApJ* **612**, 276 (2004)]

What kind of distribution function?

Fluid Equations – the Chapman-Enskog expansion

Normal criterion for using expansion:
$$\begin{cases} \epsilon = \dfrac{\lambda_{mfp}}{L} \ll 1 \\ \dfrac{\partial \ln f}{\partial t} \ll \nu \text{ Collision rate/frequency} \end{cases}$$

Fokker-Planck Equation:
$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = \frac{df}{dt} = C(f, f)$$

$\mathcal{O}(\epsilon \nu f)$ $\mathcal{O}(\nu f)$

$$f(\mathbf{r}, \mathbf{v}, t) = f_0(\mathbf{r}, \mathbf{v}, t) + \epsilon f_1(\mathbf{r}, \mathbf{v}, t) \ldots.$$

Fluid Equations – the Chapamn-Enskog expansion I

0'th order

$$C(f_0, f_0) = 0$$

$$\rightarrow f_0 = \left(\frac{n}{\pi^{3/2} v_{th}^3}\right) e^{-\left(\frac{(\mathbf{v}-\mathbf{V})^2}{v_{th}^2}\right)}$$

Boltzmann's H theorem – this is the unique solution

Fluid Temperature

$$n = n(\mathbf{r}, t) \quad v_{th} = \left(\frac{2T(\mathbf{r}, t)}{m}\right)^{1/2} \quad \mathbf{V} = \mathbf{V}(\mathbf{r}, t)$$

Fluid Density

Fluid Velocity

How do these Moments evolve?

Fluid Equations – the Chapman-Enskog expansion

$\boxed{\text{1st order}}$

$$\frac{df_0}{dt} = C(\epsilon f_1, f_0) + C(f_0, \epsilon f_1))$$

In general

$$f_{eq}(\mathbf{v},t) = \underbrace{\frac{n_0}{v_{th}^3 \pi^{3/2}} e^{-x^2}}_{f_0} \left(1 + \underbrace{[a_0(x) + a_1(x)\mathbf{x}\cdot\mathbf{b}_1 + a_2(x)\mathbf{x}\cdot\mathbf{b}_2 + a_3(x)\mathbf{x}\cdot\mathbf{M}\cdot\mathbf{x}]}_{\epsilon f_1}\right)$$

$$\mathbf{x} = \frac{\mathbf{v} - \mathbf{V}(\mathbf{r},t)}{v_{th}}$$

Compute moments to find heat flow (thermal conductivity), momentum flow (viscosity), resistivity etc.. FLUID EQUATIONS CLOSED

Finding explicit expressions for $a_0(x), a_1(x), a_2(x)$ and $a_4(x)$ is hard – it is usually done by expanding in Sonine (Laguerre) polynomials.

Unmagnetized electron Case

unmagnetized plasma where $\rho_e \gg \lambda_{mfp}$:

$$\mathbf{b}_1 = (x^2 - \frac{5}{2})\nabla \ln T_e$$

$$\mathbf{b}_2 = \frac{\mathbf{R_e}}{p_e} + \frac{m_e \nu_{ei} \mathbf{u}}{T_e}$$

$$\mathbf{M} = \mathbf{W} = \boldsymbol{\nabla}\mathbf{V_e} + (\boldsymbol{\nabla}\mathbf{V_e})^T - \frac{2}{3}(\boldsymbol{\nabla} \cdot \mathbf{V_e})\mathbf{I}.$$

Symmetric traceless
Rate of strain tensor

Fluid Equations – Braginskii 1958

SI Braginskii *Reviews of Plasma Physics, Vol. 1*. English translation 1965

$$\frac{d^\alpha n_\alpha}{dt} + n_\alpha \nabla \cdot \mathbf{v}_\alpha = 0;$$

$$m_\alpha n_\alpha \frac{d^\alpha \mathbf{v}_\alpha}{dt} = -\nabla p_\alpha - \nabla \cdot P_\alpha + Z_\alpha e n_\alpha \left[\mathbf{E} + \frac{1}{c}\mathbf{v}_\alpha \times \mathbf{B}\right] + \mathbf{R}_\alpha;$$

$$\frac{3}{2} n_\alpha \frac{d^\alpha k T_\alpha}{dt} + p_\alpha \nabla \cdot \mathbf{v}_\alpha = -\nabla \cdot \mathbf{q}_\alpha - P_\alpha : \nabla \mathbf{v}_\alpha + Q_\alpha.$$

Closure terms like **q, P,** Q come from f_1

For example electron heat flux

$$\mathbf{q}_e(\mathbf{r}, t) = \int d^3\mathbf{v} \frac{1}{2} m_e (\mathbf{v} - \mathbf{V_e})^2 (\mathbf{v} - \mathbf{V_e}) f_{e1}$$

$$= -\frac{3.2 n_e T_e}{\nu_e m_e} \nabla T_e$$

But, is the solution stable? Well posed?

Obviously the fluid equations can describe instabilities (e.g. MHD instabilities). This does not violate the assumptions behind Chapman-Enskog theory.

BUT: Is the distribution function: $f(\mathbf{r}, \mathbf{v}, t) = f_0(\mathbf{r}, \mathbf{v}, t) + \epsilon f_1(\mathbf{r}, \mathbf{v}, t)....$

stable t

If it is unstable at these scales then Chapman-Enskog solution for f_1 is incorrect "**Ill-posed**"

$$f(\mathbf{r}, \mathbf{v}, t) = f_0(\mathbf{r}, \mathbf{v}, t) + \epsilon f_1(\mathbf{r}, \mathbf{v}, t) + \delta \bar{f}(\mathbf{v}) e^{\gamma t + i\mathbf{k}\cdot\mathbf{r}}$$

$$\mathbf{E} = \delta \mathbf{E} e^{\gamma t + i\mathbf{k}\cdot\mathbf{r}}, \qquad \mathbf{B} = \delta \mathbf{B} e^{\gamma t + i\mathbf{k}\cdot\mathbf{r}}$$

Homogeneous Plasma nearly Maxwellian

At these scales and timescales f_0 and f_1 are Homogeneous and stationary in time.
Obviously f_0 is stable – it's a Maxwellian – Landau damped modes.

How big must epsilon be to drive instability? Consider electrons at this point.

$$(\gamma + i\mathbf{k}\cdot\mathbf{v})\delta f = \frac{q}{m}\left(\delta\mathbf{E}\cdot\frac{\partial(f_0+\epsilon f_1)}{\partial\mathbf{v}} + \mathbf{v}\times\delta\mathbf{B}\cdot\frac{\partial(\epsilon f_1)}{\partial\mathbf{v}}\right)$$

To make f_1 terms compete we must have $\delta\mathbf{E}\sim\epsilon v_{th}\delta\mathbf{B}$ *i.e.* Electromagnetic

Faraday gives $\delta\mathbf{B} = \frac{-i}{\gamma}\mathbf{k}\times\delta\mathbf{E}$ combining $\gamma\sim\epsilon k v_{th}$

Form of Weibel instability

Lots of Algebra

$$\delta \mathbf{B} = \frac{-i}{\gamma}\mathbf{k} \times \delta \mathbf{E}$$

$$\nabla \times \delta \mathbf{B} = \mu_0 \delta \mathbf{J} = -e\mu_0 \int \mathbf{v} \delta f d^3\mathbf{v}$$

$$\delta f = \frac{-e}{(i\mathbf{k}\cdot\mathbf{v})m_e}[\delta \mathbf{E} \cdot \frac{\partial f_0}{\partial \mathbf{v}} - \frac{i\epsilon}{\gamma}\mathbf{v} \times (\mathbf{k} \times \delta \mathbf{E}) \cdot \frac{\partial f_1}{\partial \mathbf{v}}]$$

$$f_{eq}(\mathbf{v},t) = \frac{n_0}{v_{th}^3 \pi^{3/2}} e^{-x^2} \overset{f_0}{(1} + \overbrace{[a_0(x) + a_1(x)\mathbf{x}\cdot\mathbf{b}_1 + a_2(x)\mathbf{x}\cdot\mathbf{b}_2 + a_3(x)\mathbf{x}\cdot\mathbf{M}\cdot\mathbf{x}]}^{\epsilon f_1})$$

Dispersion Relation

Electric field is perpendicular to **k** and satisfies

$$\delta\mathbf{E}_\perp \cdot \left[\mathbf{M} - \mathbf{M}\cdot\hat{\mathbf{k}}\hat{\mathbf{k}}\right] = \Lambda\delta\mathbf{E}_\perp.$$

two normalized eigenvectors $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$, with eigenvalues Λ_1 and Λ_2

$$d = \frac{c}{\omega_{pe}} = collisionless\ \ skin\ \ depth$$

Growth rate

$$\gamma_j = \pi^{-1/2} k v_{th}\{A_{3B}\left[\hat{\mathbf{k}}\cdot\mathbf{M}\cdot\hat{\mathbf{k}} - \hat{\mathbf{e}}_j\cdot\mathbf{M}\cdot\hat{\mathbf{e}}_j\right] - k^2d^2 + i[A_{1B}(\hat{\mathbf{k}}\cdot\mathbf{b_1}) + A_{2B}(\hat{\mathbf{k}}\cdot\mathbf{b_2})]\}.$$

$$A_{3B} = \frac{4}{3\pi^{1/2}}\int_0^\infty dx x^4 e^{-x^2} a_3(x) \sim \frac{1}{\nu_e}$$

$$A_{1B} = \pi^{1/2}\int_0^\infty dx x^3 e^{-x^2} a_1(x)$$

A_{2B} is similar

Growth Rate

Optimal direction is when **k** is in direction of maximum eigenvalue of **M.**

$$\mathbf{M}\cdot\hat{\mathbf{u}} = \lambda\hat{\mathbf{u}}$$

eigenvectors $\hat{\mathbf{u}}_1$, $\hat{\mathbf{u}}_2$ and $\hat{\mathbf{u}}_3$ with eigenvalues (respectively) λ_1, λ_2 and λ_3

$$\lambda_1+\lambda_2+\lambda_3=0. \quad \lambda_1>\lambda_2>\lambda_3$$

$$\gamma_{optimum} = \pi^{-1/2}kv_{th}\{A_{3B}\left[\lambda_1-\lambda_3\right]-k^2d^2+i[A_{1B}(\hat{\mathbf{k}}\cdot\mathbf{b_1})+A_{2B}(\hat{\mathbf{k}}\cdot\mathbf{b_2})]\}.$$

$$\boxed{\gamma_{max} = \frac{2}{\sqrt{\pi}}\frac{v_{th}}{d}\epsilon^{3/2}.}$$

$$k_{max} = (1/d)\sqrt{3\epsilon}.$$

$$\epsilon = \frac{1}{3}A_{3B}\left[\lambda_1-\lambda_3\right]$$

Instability driven by flow shear not heat flux.

Criterion for Existance

Instability exists if it can be
at collisionless scales:

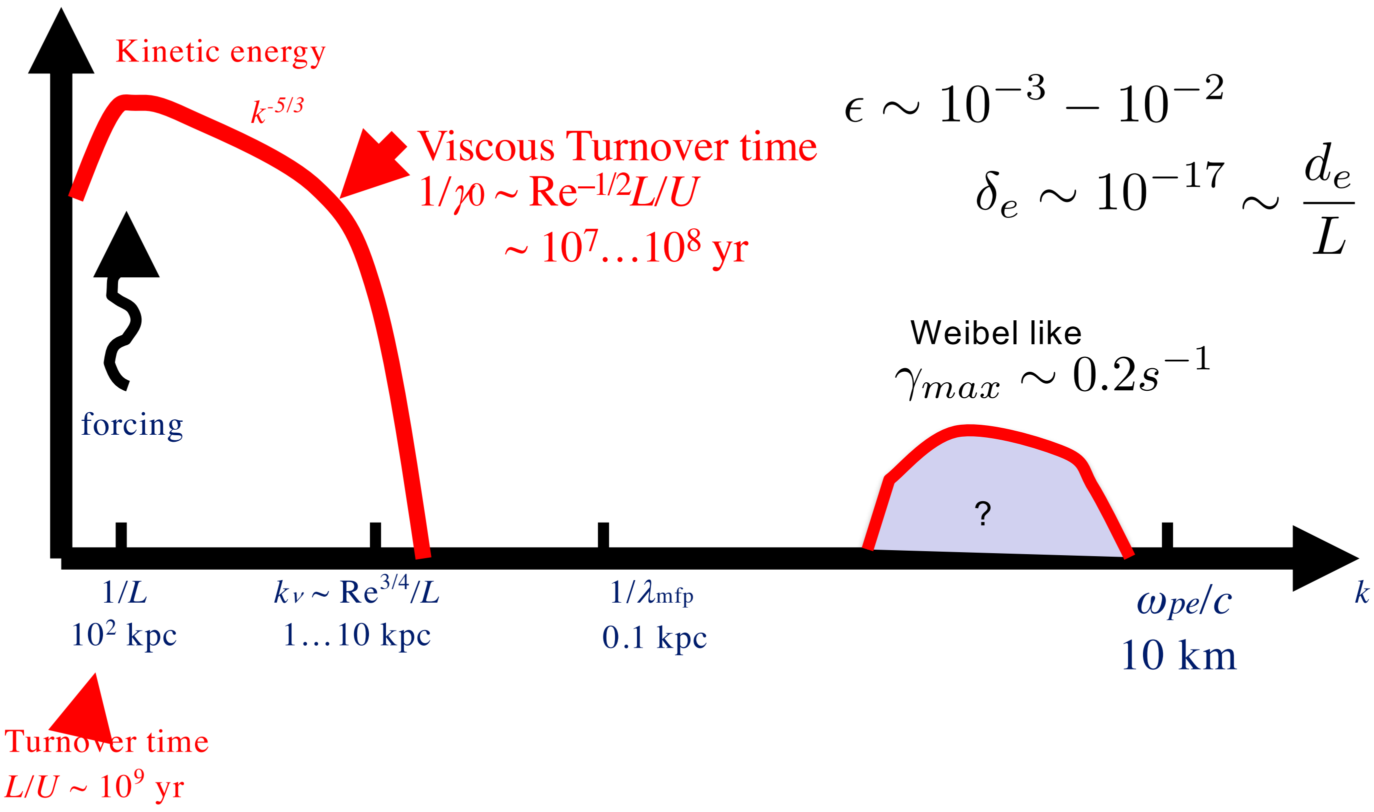
$$k_{max}\lambda_{mfp} \gg 1$$

$$\rightarrow \frac{\lambda_{mfp}}{d}\sqrt{3\epsilon} \gg 1$$

Cluster $\epsilon \sim \sqrt{\frac{m_e}{m_i}} M^{3/2} \left(\frac{\lambda_{mfp}}{L_0}\right)^{1/2} \sim 10^{-3} - 10^{-4}$

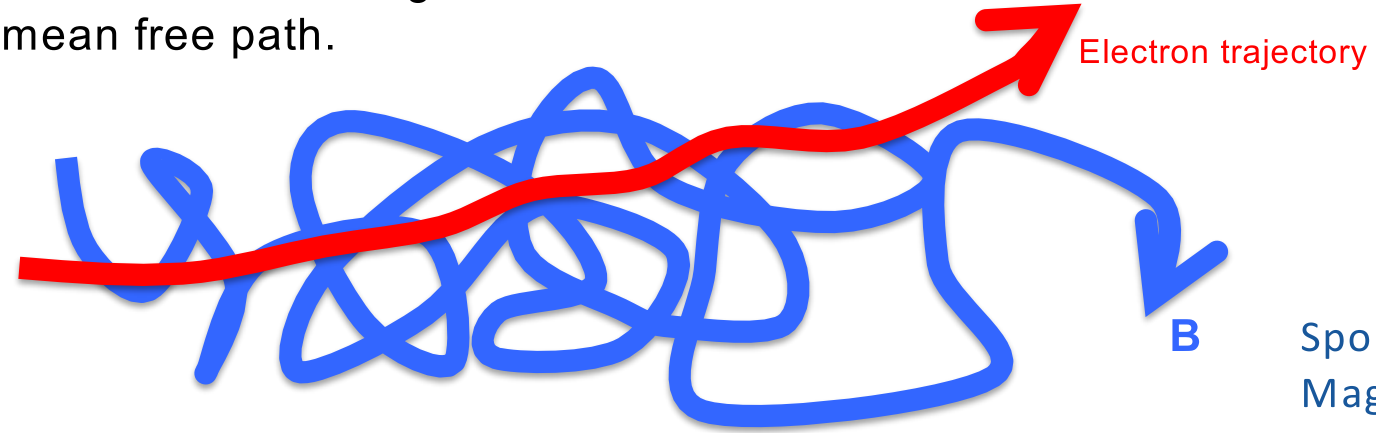
$$\frac{\lambda_{mfp}}{d} \sim 3 \times 10^{14}.$$

Unmagnetised



Nonlinear Effect - Anomalous scattering

Increased scattering? Decrease the effective mean free path.



Spontaneous Magnetization From instability

$$\Delta \mathbf{v} \sim \frac{ev\delta B}{m_e}\tau \sim \Omega_c \tau v$$

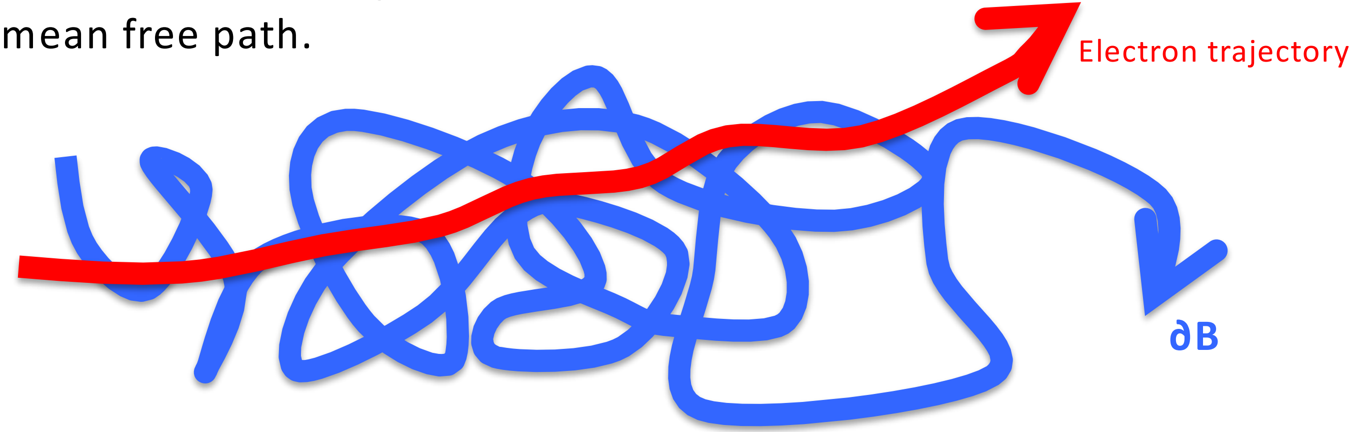
Deflection in correlation time

$$\tau \sim (kv)^{-1}$$

Correlation time

Anomalous scattering. Estimates

Increased scattering? Decrease the effective mean free path.



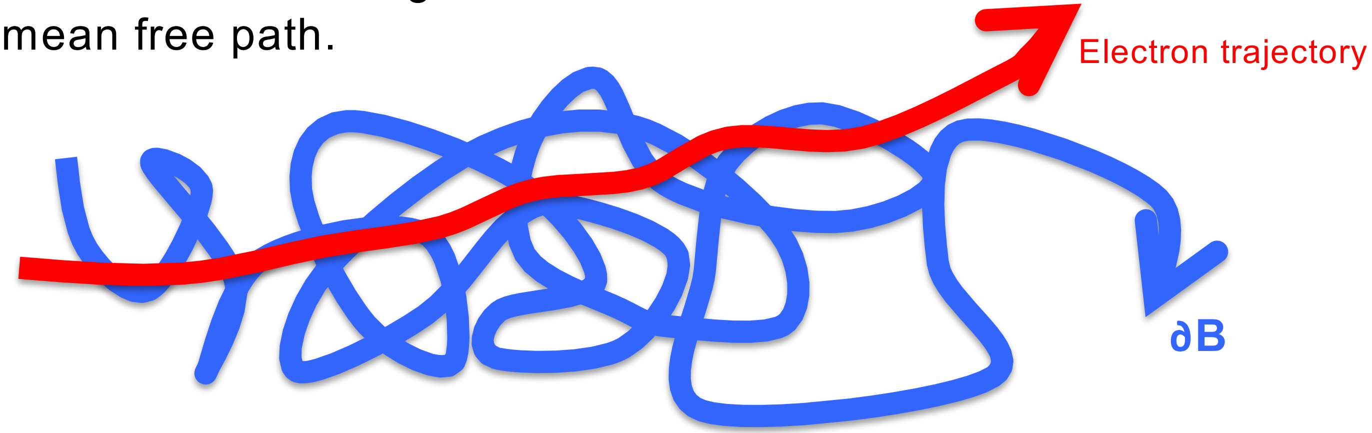
$$D_{Bv} \sim \frac{\Delta \mathbf{v}^2}{\tau} \sim \frac{\Omega_c^2 v}{k}$$ Diffusion in velocity

Anomalous scattering rate $$\nu_{anom} \sim \frac{D_{Bv}}{v^2} \sim \frac{\Omega_c^2}{kv}$$

$$\boxed{\nu_{eff} \sim \nu_c + \nu_{anom}}$$

Nonlinearity

Increased scattering? Decrease the effective mean free path.



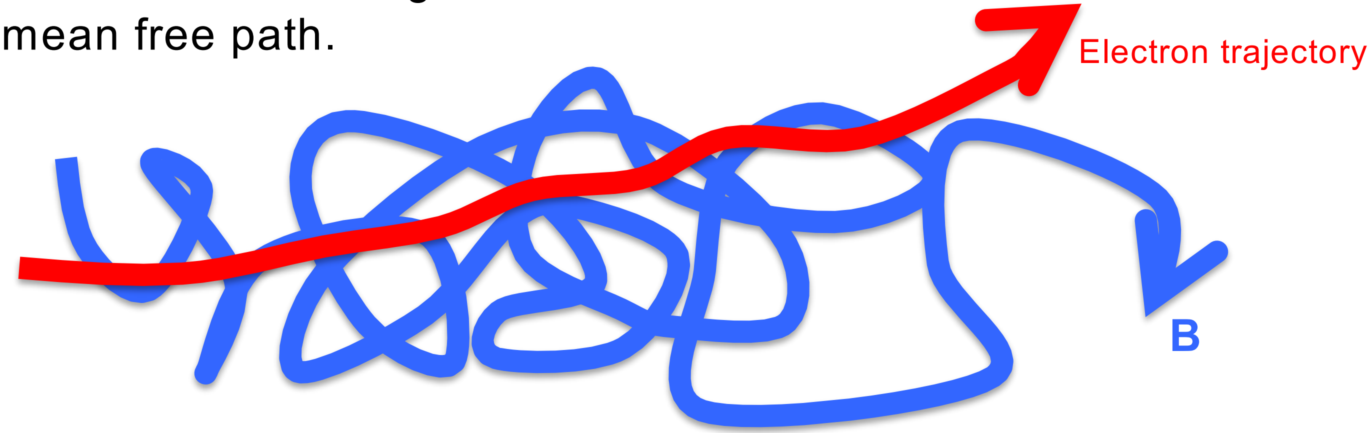
$$\nu_c \sim \nu_{anom}$$ Beginning of nonlinear behaviour

$$\rho_{crit} = \frac{v_{the}}{\Omega_c} \sim \epsilon^{-1/4}\sqrt{(d\lambda_{m.f.p.})}$$

$$f_1 \rightarrow f_1 \frac{\nu_c}{\nu_{anom}}$$

Nonlinearity

Increased scattering? Decrease the effective mean free path.



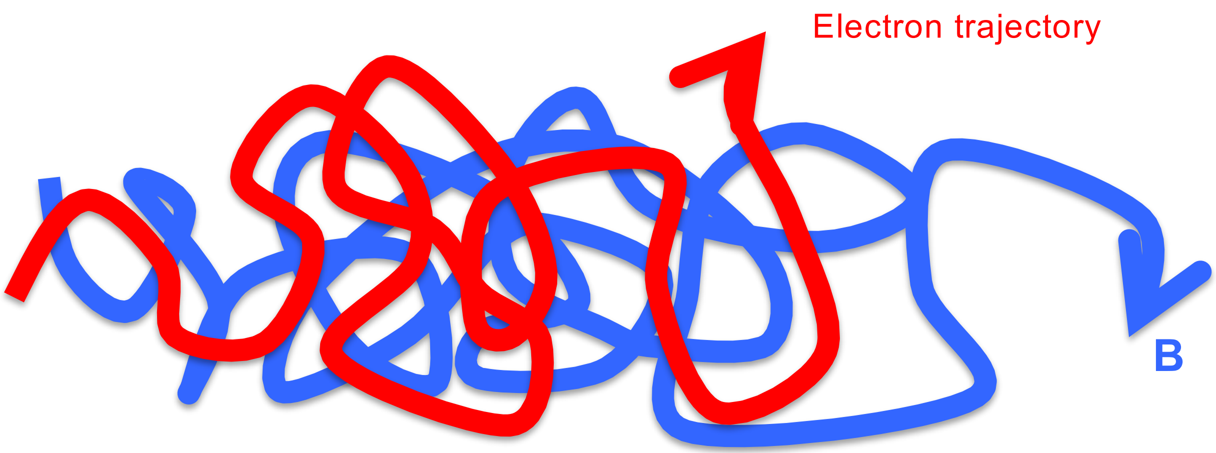
Modified growth rate

$$\gamma = \pi^{-1/2} k v_{th} \{ \epsilon \frac{\nu_c}{\nu_{eff}} - k^2 d^2 \} \sim \pi^{-1/2} k v_{th} \{ \epsilon \frac{k \rho^2}{\lambda_{mfp}} - k^2 d^2 \}.$$

When $\nu_{anom} \gg \nu_c$

Saturation when $k_{max} \rho \sim 1 \rightarrow \rho \sim \epsilon^{-1/3} (\frac{3}{2} d^2 \lambda_{mfp})^{1/3}.$ $\beta_{sat} \sim \epsilon^{-2/3} \left(\frac{\lambda_{mfp}}{d} \right)^{2/3}$

Saturation



Effective mean free path in saturation $\rho \sim \epsilon^{-1/3}(\frac{3}{2}d^2\lambda_{mfp})^{1/3}.$

cluster

Thermal conductivity $\kappa_e \sim \rho v_{th} \sim \epsilon^{-1/3}\left(\frac{d}{\lambda_{mfp}}\right)^{2/3}\kappa_0 \sim 10^{-10}\kappa_0$

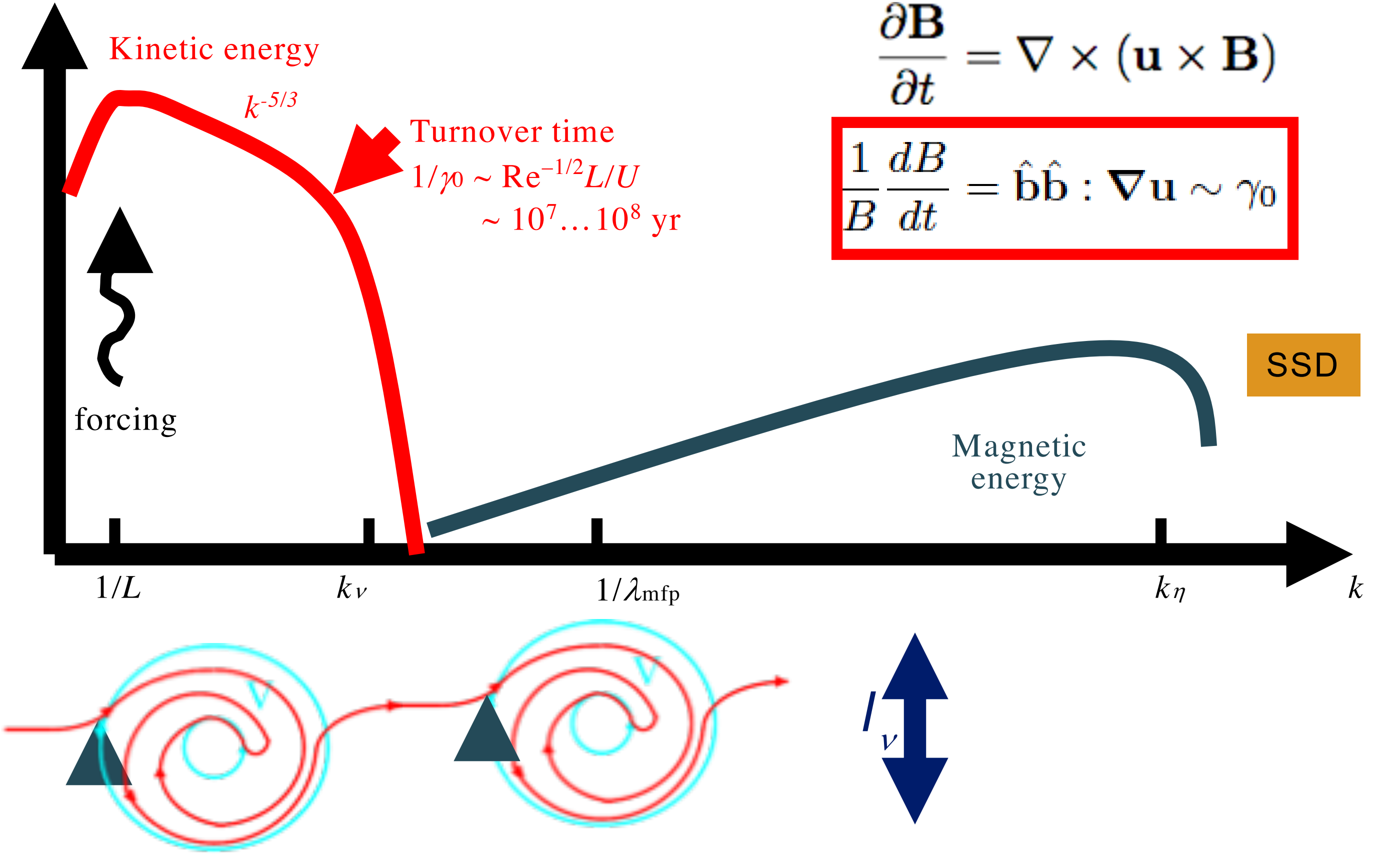
Spitzer conductivity $\kappa_0 \sim \lambda_{mfp} v_{th}$

Field keeps on Growing

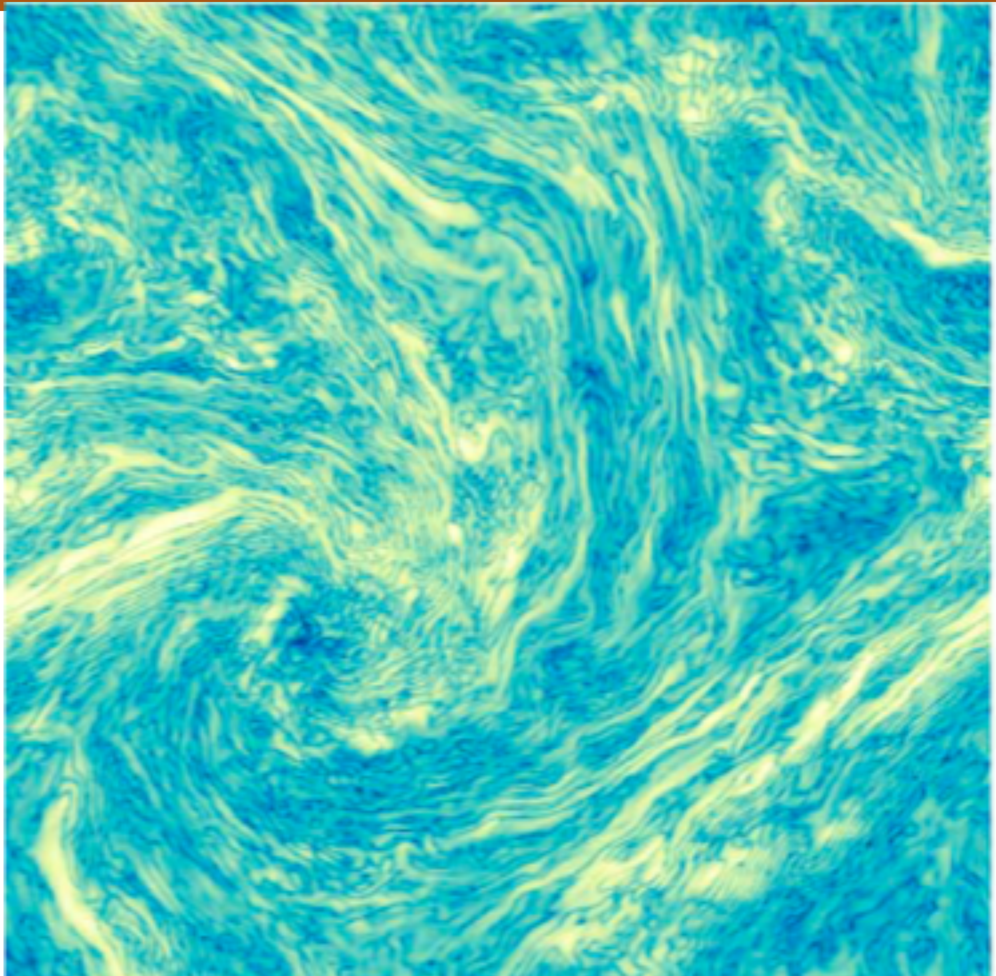
Stretching and Amplifying the Field by the turbulence

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\boxed{\frac{1}{B}\frac{dB}{dt} = \hat{\mathbf{b}}\hat{\mathbf{b}} : \boldsymbol{\nabla}\mathbf{u} \sim \gamma_0}$$



What to do about the small scale stuff? Simulation



St-Onge and Kunz

Collisionless dynamo simulation

Not enough resolution to do electron Weibel scales

What to do about the small scale stuff? Speculation

Mean field theories? Effective transport?

$$
\begin{aligned}
\mathbf{q}(\mathbf{r},t) &= \mathbf{q}(\nabla n, \nabla T, \mathbf{W}, \mathbf{B}, T, n...)\\
\mathbf{\Pi}(\mathbf{r},t) &= \mathbf{\Pi}(\nabla n, \nabla T, \mathbf{W}, \mathbf{B}, T, n...)\\
\sigma(\mathbf{r},t) &= \sigma(\nabla n, \nabla T, \mathbf{W}, \mathbf{B}, T, n...)
\end{aligned}
$$

Compute these effective transport coefficients – very tough to do because epsilon must be small. This would be a tough thesis.

Thank You