LASER-AIDED PLASMA DIAGNOSTICS: A BRIEF OVERVIEW

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OUTLINE

➤ Brief definition of lasers

➤ Laser as a tool for plasma diagnostics
  - Ion velocity distribution function via LIF
  - Laser for plasma interferometry
  - Laser scattering as a tool to probe the plasma

➤ Summary
In 1917 Einstein introduced a model of the interaction of radiation and matter.

**absorption:**

an incident photon is absorbed while the atomic state is elevated from energy level $E_1$ to $E_2$

**spontaneous emission:**

a photon is emitted while the atomic system descends from energy level $E_2$ to $E_1$

**stimulated emission:**

an additional photon is emitted when an atomic system is under the action of an incident photon→ light amplification / LASER
LIGHT AMPLIFICATION IS NOT COMMON IN NATURE

➤ Thermal equilibrium between light and matter
  - Absorption of light is much more dominant over gain because of the Boltzmann’s distribution of matter.

➤ Probability for transitions resulting in emission at optical frequencies at room temperature is about $e^{-40}$

For a good theoretical description of the conditions for optical gain/light amplification see Simon Hooker and Colin Webb in Laser Physics
Population per state must be greater in the upper state than in the lower state: this is called population inversion.

- **Condition for the required population inversion:**
  - Selective pumping: upper level is pumped more rapidly than lower level
  - Favorable lifetime ratio
  - Favorable degeneracy

- **Practical:** Need some sort of external pump to raise atomic population from lower to upper energy level
EXAMPLE OF PUMPING

Examples of crystals?
OUTLINE

➤ Brief definition of lasers

➤ Laser as a tool for plasma diagnostics
  - Ion velocity distribution function via Laser-Induced Fluorescence
  - Laser for plasma interferometry
  - Laser scattering as a tool to probe the plasma

➤ Summary
LASER-INDUCED-FLUORESCENCE (LIF)

Optical diagnostic technique that provides local measurements of the velocity distribution function (ions or neutrals).

Knowledge of distribution function $f(x,v,t)$ for each species can help understand phenomena:

- Vlasov equation
- Landau damping
- Ion heating by waves
- Other moments can be determined.
Assume the laser beam propagating through a plasma

\[ \mathcal{W}(v) = \frac{\lambda^2 I_0}{8\pi \hbar \omega_0} \gamma \int d\omega \Phi(\omega) L(\omega - \mathbf{k}_{\text{laser}} \cdot \mathbf{v}) \text{ [photons/s]} \]

Where \[ L(\omega) = \frac{1}{2\pi} \frac{\gamma^2}{(\omega - \omega_0)^2 + \left(\frac{\gamma}{2}\right)^2} \]
PRINCIPLE AND GOVERNING EQUATIONS

Assume the laser beam propagating through a plasma.

The absorption rate can be described:

\[ \mathcal{W}(\mathbf{v}) = \frac{\lambda_0^2 I_0}{8\pi h\omega_0} \gamma \int d\omega \Phi(\omega) L(\omega - \mathbf{k}_{\text{laser}} \cdot \mathbf{v}) \] [photons/s]

Where

\[ L(\omega) = \frac{1}{2\pi} \frac{\gamma^2}{(\omega - \omega_0)^2 + (\frac{\gamma}{2})^2} \]

The velocity distribution can in principle be extracted by sweeping the laser wavelength!

\[ \frac{dN}{d\Omega} = \frac{\gamma I_0}{8\pi h\omega_0} \lambda^2 f_{k_{\text{laser}}} \left( \frac{\omega_{\text{laser}} - \omega_0}{k_{\text{laser}}} \right) \] #Photons/s/solide angle

Moderate pumping

\[ f_2(\mathbf{v}) = \frac{\mathcal{W}(\mathbf{v})}{\gamma} f_1(\mathbf{v}) \]

Ion velocity distribution of the excited state
In practice, a three-level system is preferred: facilitate the discrimination between pump and induced fluorescence.

- Analysis requires the rate equation for all three levels.

- LIF approach is limited to low temperature plasmas with spectral lines accessible with commercial laser.
EXAMPLE OF MEASURED AR II DISTRIBUTION FUNCTION (IVDF)

➤ Measurements are performed in a linearly magnetized Ar II plasma

![Energy level diagram and graph showing measured distribution function (IVDF) with calculated thermal velocity $V_{thi} = 536.65 \text{m/s}$]
Understand the contribution of ubiquitous coherent instabilities to cross-field transport of thrusters.
We mix the above with the modulation of the laser beam ($\omega_{\text{laser}}$), and observed for each ion velocity class and spatial location:

$$f(t, x, v) = f^0(x, v) + \sum_{n>0} f^n(x, v) \sin(n\omega_D t + \theta_n(x, v))$$

- Time-averaged IVDF
- Breathing mode

\[
\begin{align*}
    f^0(x, v) &\rightarrow \omega_{\text{laser}} \\
    f^1(x, v) &\rightarrow \omega_{\text{laser}} \pm \omega_D \\
    f^n(x, v) &\rightarrow \omega_{\text{laser}} \pm n\omega_D \\
\end{align*}
\]

... and the IVDF(t) can be reconstructed!
HARMONIC BASED RECONSTRUCTION OF THE TIME-RESOLVED IVDF (WITH UP TO N=1)

Reconstructed time-dependent IVDF

Diallo et al., RSI 86(3):033506, 2015.
Optical tagging enables to track the evolution of a small volume in phase space.
OTHER LIF APPLICATIONS

➤ Using the Stark effect, one can probe the local electric field:
  - caveat: this is possible if the various broadening mechanisms are small.
  ➤ Please describe the various broadening mechanisms?
  - To circumvent the broadening issues, we proposed a method probing the Rydberg state directly.


➤ In certain cases (weak field), LIF can provide measurements of the local magnetic field via Zeeman effect.
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➤ Summary
BASICS OF LASER INTERFEROMETRY

➤ Relies on measurements of plasma optical refractivity -
  - The index of refraction is given:

\[
n = \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \approx 1 - \frac{1}{2} \frac{\omega_p^2}{\omega^2}
\]

➤ Observations of the shift of the interference fringes with and without intervening gas

\[
\Delta \phi = L(n - 1)/\lambda \Rightarrow -4.46 \times 10^{-14} n_e L \lambda
\]
THINGS TO CONSIDER

- Good interferometric techniques should allow fringe shifts of $1/100 \lambda$
  - Example: for $\lambda = 500$ nm, a minimum detectable electron density is about $10^{16}$ electrons/cm$^3$

- What if there are non-electronic contributions to the index of refraction?

  \[ n - 1 = 2\pi \alpha n_a \]
  \[ \lambda_1 \Delta \phi_1 = \left[ -\frac{1}{2} \frac{\nu^2}{c^2} + (n_a - 1) \right] L \]

- Electronic effects can be separated from atomic effects by making fringe-shift measurements at two different wavelengths (can be also used to separate mechanic vibrations).

  \[ \lambda_1 \Delta \phi_1 - \lambda_1 \Delta \phi_2 = -\frac{1}{2} \frac{\nu^2}{c^2} L \left[ \lambda_1^2 - \lambda_2^2 \right] \]
OUTLINE

➤ Brief definition of lasers

➤ Laser as a tool for plasma diagnostics
  - Ion velocity distribution function via LIF
  - Laser for plasma interferometry
  - Laser scattering as a tool to probe the plasma: Thomson scattering

➤ Summary
The scattering of laser radiation by plasma particles is an extremely important tool in the diagnosis of plasmas and in studying microscopic fluctuation phenomena occurring in plasmas.

Important for characterizing the plasmas through the determination of electron and ion temperatures and the electron density.

Important to the kinetic theory of a system of interacting, charged particles by characterizing instabilities in most laboratory plasmas.
THOMSON SCATTERING: WHAT IS IT ALL ABOUT?

Sir J.J Thomson 1856-1940

English physicist and Nobel laureate in physics, credited with the discovery and identification of the electron; and with the discovery of the first subatomic particle.

Plasma in scattering volume

Scattered spectrum (hence; temperature of the plasma is known)
THOMSON SCATTERING (TS) APPROACH

➤ Thomson scattering is a powerful and non-perturbing diagnostic technique.
➤ It provides detailed information about the electron density and temperature.

TS provides direct and localized measurements of electrons properties

Prunty Phys. Scr. 89 128001 (2014)
GENERAL THOMSON SCATTERING SCHEME
RELATIVISTIC THOMSON SCATTERING - ESTIMATES FOR ITER

What is the effect of the laser field on the single electron?

\[ E \text{ and } B : \quad E_{\text{laser}} = 5 \text{ J} @ 200 \text{ ps} \rightarrow 25 \text{ GW} \]

Beam diameter = 5 cm \( \rightarrow \) Poynting vector is \( S = E \times H \)

\[ S = \frac{1}{2} \epsilon_0 c E^2 \rightarrow E \approx 10^8 \text{V/m} \text{ and } B = \frac{E}{c} \approx 0.3 \text{T} \]

**Typical velocity acquired by the electron in the field of the light wave**

\[ v = \frac{eE}{m_0 \omega} \sin(\omega t) \approx 3 \times 10^4 \text{m/s} \text{ assuming } \lambda_i = 1064 \text{ nm} \]

Let’s estimate the thermal velocity of the electron for ITER?
RELATIVISTIC THOMSON SCATTERING - ESTIMATES FOR ITER

\[ T_e = 40 \text{ keV} : v = 1.2 \times 10^8 \text{ m/s} \rightarrow \beta = \frac{v}{c} \sim 0.4 \]

The B-field of the light wave cannot be neglected.

\[ \mathbf{E} \text{ and } \mathbf{B} : \quad \mathbf{E}_{\text{laser}} = 5 \text{ J @ 200 ps} \rightarrow 25 \text{ GW} \]

Beam diameter = 5 cm \rightarrow \text{Poynting vector is } \mathbf{S} = \mathbf{E} \times \mathbf{H} \quad \boxed{S = 1.27 \times 10^{13} \text{W/m}^2}

\[ S = \frac{1}{2} \epsilon_0 c E^2 \rightarrow E \approx 10^8 \text{V/m} \text{ and } B = \frac{E}{c} \approx 0.3 \text{T} \]

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\[ v = \frac{eE}{m_0 \omega} \sin(\omega t) \approx 3 \times 10^4 \text{m/s assuming } \lambda_i = 1064 \text{ nm} \]

This is considerably less than the actual electron velocities, so the laser beam does not influence the electron motion (“unperturbed electron velocity approximation”).
**APPROACH TO DETERMINE THE SPECTRUM**

Equation of motion for an electron in field of light wave (laser):

\[
\frac{d}{dt} \left[ \frac{m_0 v}{\sqrt{1-v^2/c^2}} \right] = -e \left( E_i + v \times B_i \right)
\]

\[\partial_t \beta\]

Scattered electric field:

\[
E_s = -\frac{e}{4\pi\epsilon_0} \left[ \frac{1}{(1-\beta \cdot \hat{s})^3 Rc} \hat{s} \times (\hat{s} - \beta) \times \partial_t \beta \right]_{\text{retarded}}
\]

Lienard-Wiechert potentials for a moving charge

Thomson spectrum!

Coordinate system

Velocity distribution:

\[
f(\beta) = \frac{\alpha}{2\pi K_2(2\alpha)} \frac{\exp(-2\alpha(1-\beta^2)-1/2)}{(1-\beta^2)^{5/2}}
\]

\[
\alpha = m_0 c^2 / 2kT
\]

\[
K_2(2\alpha) \text{ is the modified Bessel function of second order and second kind.}
\]

Prunty Phys. Scr. 89 (2014) 128001
the observation wave vector properties are defined by the incident wave vector and direction of detection

\[ \mathbf{k} = \mathbf{k}_s - \mathbf{k}_i \]
\[ \omega = \omega_s - \omega_i \]

Bragg relation

\[ |k| \approx 2 |k_i| \sin(\theta/2) \]
SINGLE ELECTRON SCATTERING

- Power per unit solid angle scattered by electron

\[
\frac{dP}{d\Omega} = r_e^2 \sin^2 \phi \ c\varepsilon_0 \ |E_i|^2
\]

- It is common to define a **differential cross-section ratio** of scattered power to incident power per unit area

\[
\frac{d\sigma}{d\Omega} = r_e^2 \sin^2 \phi
\]

It represents the total Thomson scattering cross-section for an electron:

\[
\sigma = \frac{8\pi}{3} r_e^2
\]

with

\[
r_e = 2.82 \times 10^{-15} \text{ m}
\]
THOMSON SCATTERED POWER

General form of the Thomson scattered power per unit solid angle, a distance $R$ from the scattering volume:

$$\frac{dP}{d\Omega} = \frac{cR^2}{4\pi} \left( \sum_{j=1}^{N} E_{js} \sum_{l=1}^{N} E_{ls} \right)$$

$N =$ number of particles

$E_s =$ scattered electric field

or, separating terms into $j = l$, and $j \neq l$:

$$\frac{dP}{d\Omega} = \frac{cR^2}{4\pi} \left( \frac{N E_s^2}{2} + N(N - 1)\overline{(E_j \cdot E_l)}_{j\neq l} \right)$$

Two different scattering regimes emerge
The phase factor $k \cdot r_{po}$ depends on the electron’s location and the vector $k$.

Electrons lying on the locus defined by $k \cdot r_{po} = C$, a constant, gives rise to the same phase.

Equation of a plane whose normal is $k$ and all electron lying on this plane scatter in phase.
GEOMETRY

Observer laser

Imaginary scattering planes (normal to $k$)

Incident wavefront

electrons

\[ \vec{k} = \vec{k}_s - \vec{k}_i \]

\[ \omega_s = \omega_i + \vec{k} \cdot \vec{v} \]
IN PHASE SCATTERING

\[
\tilde{k} \cdot \left( \tilde{r}_{po} + \Delta \tilde{r}_{po} \right) = C + 2\pi
\]

Hence:

\[
|\Delta \tilde{r}_{po}| = \frac{2\pi}{|\tilde{k}|}
\]

This quantity is the scale-length for scattering; (measures resolution on which plasma events are viewed in a scattering experiment)

Now:

\[
|\tilde{k}| = \sqrt{\tilde{k} \cdot \tilde{k}} = \sqrt{\tilde{k}_s \cdot \tilde{k}_i \cdot (\tilde{k}_s - \tilde{k}_i) \cdot (\tilde{k}_s - \tilde{k}_i)}
\]

\[
= \sqrt{k_s^2 + k_i^2 - 2k_s k_i \cos \theta}
\]

\(\theta\) is the scattering angle.

Hence:

\[
|\tilde{k}| = \frac{1}{c} \sqrt{\omega_s^2 + \omega_i^2 - 2\omega_s \omega_i \cos \theta}
\]

As \(\omega_s \approx \omega_i\)

\[
|\tilde{k}| = \frac{1}{c} \sqrt{2\omega_i^2 - 2\omega_i^2 \cos \theta} = \frac{\omega_i \sqrt{2}}{c} \sqrt{1 - \cos \theta}
\]

therefore;

\[
|\tilde{k}| = \frac{2\omega_i}{c} \sin \frac{\theta}{2} = \frac{4\pi}{\lambda_i} \sin \frac{\theta}{2}
\]

\(\omega_s = \omega_i + \tilde{k} \cdot \tilde{v}\)
The scale length for scattering is $2\pi/k$ - resolution.

Correlated interactions between the plasma electrons only occur at or above a certain scale length - so called Debye length $\lambda_D$.

\[ \lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{e^2 n_e}} \]

\[ \alpha = \frac{1}{k \lambda_D} \ll 1 \]

\[ \alpha = \frac{1}{k \lambda_D} > 1 \]

Which one represents the incoherent scattering?
Incoherent scattering: random distribution of particles \( \rightarrow \) phases add up destructively

\[
\frac{dP}{d\Omega} = \frac{c R^2}{4\pi} \left( \frac{N E_s^2}{2} + N(N - 1) \langle E_j \cdot E_l \rangle_{j \neq l} \right)
\]
INFORMATION FROM INCOHERENT THOMSON SCATTERING

Scattered power

\[ P_s = P_i n_e \Delta L \Omega \frac{d\sigma_T}{d\Omega} S(k, \omega) \]

Injected power

Electron density

spectral density function or form factor

\[ S(k, \omega) = \int_{-\infty}^{+\infty} F_k(v_k) \delta[\omega_i - \omega_s(v)] dv_k \]

velocity distribution function along \( k \)

\[ F_k(v_k) = \frac{1}{c_0 \sqrt{\pi}} \exp \left[ - \left( \frac{v_k}{a} \right)^2 \right] \]

where \[ c_0 = \sqrt{\frac{2k_B T_e}{m_e}} \]
INFORMATION FROM INCOHERENT THOMSON SCATTERING

Scattered power

\[ P_s = P_i n_e \Delta L \Omega \frac{d\sigma_T}{d\Omega} S(k, \omega) \]

\[ S(k, \omega) = \int_{-\infty}^{+\infty} F_k(v_k) \delta[\omega_i - \omega_s(v)] dv_k \]

\[ F_k(v_k) = \frac{1}{c_0 \sqrt{\pi}} \exp\left[-\left(v_k/a\right)^2\right] \]

where \( c_0 = \sqrt{\frac{2k_B T_e}{m_e}} \)

\[ k = k_s - k_i \]

\[ \omega = \omega_s - \omega_i \]

scattered spectrum area proportional to \( n_e \)

spectrum width proportional to \( \sqrt{T_e} \)
BREAKTHROUGH FOR TOKAMAKS WAS DEMONSTRATED USING THOMSON SCATTERING


Drawing from the talk “Evolution of the Tokamak” given in 1988 by B.B. Kadomtsev at Culham.
THOMSON SCATTERING SYSTEM ON NSTX-U

Beam dump

Laser flight tubes & folding mirrors

Measurements zone

Collection optics
PROFILES FOR THE LAST NSTX-U CAMPAIGN

Temperature

Density
INCOHERENT VS COHERENT THOMSON SCATTERING

- The scale length for scattering is $2\pi/k$ - resolution
- Correlated interactions between the plasma electrons only occur at or above a certain scale length - so called Debye length $\lambda_D$.

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{e^2 n_e}}$$

$$\alpha = \frac{1}{k\lambda_D} \ll 1$$

$$\alpha = \frac{1}{k\lambda_D} > 1$$
COHERENT THOMSON SCATTERING

Recall general form for scattered power:

\[
\frac{dP}{d\Omega} = \frac{cR^2}{4\pi} \left( \frac{N E_s^2}{2} + N(N-1)(E_j \cdot E_l)_{j \neq l} \right)
\]

coherent scattering: particle positions not random, instead correlated \(\rightarrow\) phases add up constructively

\[
\frac{dP}{d\Omega} = \frac{cR^2}{4\pi} \left( \frac{N E_s^2}{2} + N(N-1)(E_j \cdot E_l)_{j \neq l} \right)
\]

this term now dominates

Scattering takes place on structured "bunches" of electrons

For observation length scale is larger than the electron screening length
INFORMATION FROM COHERENT SCATTERING

Scattered power

\[ P(R, \omega_s) \ d\Omega \ d\omega_s = \frac{P \pi r_c^2}{A2\pi} \ d\Omega \ d\omega \left| \hat{s} \times (\hat{s} \times \hat{E}_{io}) \right|^2 N S(k, \omega) \]

\[ S(k, \omega) \equiv \lim_{T \to \infty, V \to \infty} \frac{1}{TV} \left\langle \frac{|n_e(k, \omega)|^2}{n_o} \right\rangle \]

\[ k = k_s - k_i \]

\[ \omega = \omega_s - \omega_i \]

- Density fluctuations visible depending on scale observed
- Electrons and ions have distinct contributions to the form factor in the collective scattering regime:
  - highly complex information depending on the plasma properties!
high frequency fluctuations linked to fast electron dynamics

low frequency fluctuations linked to slower ion dynamics

$\omega_{pe}, \omega_{pi}$: electron, ion plasma frequencies - natural oscillation frequencies

$\alpha \equiv 1/k\lambda_D$: scattering parameter ($> 1$ = coherent regime)

$Z$: ion charge

$T_e, T_i$: electron, ion temperatures

Froula, Glenzer, Luhmann and Sheffield
*Plasma scattering of electromagnetic radiation*
Acad. Press, 2nd edition
APPLICATIONS TO WARM DENSE MATTER (WDM)

WDM is an intermediate state between solids and plasmas: temperature: 1 – 100 eV, density: ~ 1 g/cm$^3$ (solid densities)

- Ions are strongly coupled
- Electrons are fully or partially degenerate

The equation of state of light elements is essential to understanding the structure of Jovian planets and inertial confinement fusion (ICF) experiments. The equation of state (EOS) in the WDM regime is largely unknown.
**X-RAY THOMSON SCATTERING (XRTS)**

Scattering vector: \[ k = (4\pi / \lambda_0) \sin(\theta/2) \]

Plasma screening length

\[ \lambda_S \sim \lambda_{TF} = \sqrt{\frac{2\varepsilon_0 E_F}{3n_e e^2}} \]  
(Thomas-Fermi length)

or \[ \lambda_{De} = \sqrt{\frac{\varepsilon_0 k_B T_e}{e^2 n_e}} \]  
(Debye length)

**Dynamic structure factor (Chihara 1987, 2000):**

\[ S^\text{tot}_{ee}(k, \omega) = |f_I(k) + q(k)|^2 S_{ii}(k, \omega) + Z_f S^0_{ee}(k, \omega) + Z_c \int \tilde{S}_{ce}(k, \omega - \omega') S_s(k, \omega') d\omega' \]

**Bound electrons/ following motion of the ions**

**Free/delocalized electrons**

\[ \alpha = 1/k\lambda_S \]
XRTS IN WARM DENSE DEUTERIUM

Rayleigh peak: elastic scattering (bound electrons)

Compton peak: inelastic scattering (free/metallic electrons)

- Determine the elements of the EOS from features of the scattering spectra:
  - \( T_e \) from width of the inelastic peak
  - \( N_e \) from the downshift of the plasmon peak
  - Ion temperature from electric scattering strength
  - Average ionization state from intensity ratio of Rayleigh and Compton peaks
  - Atomics structure from bound-free tail contribution

\[
E_C = \frac{\hbar^2 k^2}{2m_e}
\]

\[
\omega_p = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}
\]

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SUMMARY

➤ Brief description of lasers

➤ Optical diagnostic technique that provides local measurements of the ion velocity distribution function.

➤ Laser for plasma interferometry

➤ Laser scattering as a tool to probe the plasma: Incoherent vs coherent Thomson scattering and finally XRTS.
LITERATURE FOR THOMSON SCATTERING — IF YOU ARE INTERESTED

➤ Pechacek and Trivelpiece (Phys. Fluids, Vol 10, 1668 (1967))
  - first consistent treatment of relativistic Thomson scattering

  - carried out a relativistic correction to first order in v/c (15% error at 25 keV)

➤ Zhuravlev and Petrov (Sov. J. Plasma Phys., Vol 5, 3 (1979))
  - integrated the relativistic scattering integral analytically (neglecting depolarization).

  - used the analytic formula of Zhurlev and Petrov with a stated accuracy of approximately 1% up to 100 keV.

➤ Matoba et al (Jap. J. Appl. Phys., Vol. 18, No. 6, 1127 (1979))
  - derived an integral equation and approximated this by an analytic expression to second order in v/c (10% error at 25 keV)

  - derived an analytic formula using the treatment of Hutchinson (Principles of Plasma Diagnostics, Cambridge University Press 1987) with depolarization taken into account, and they went on to derive a rational approximation with high accuracy (error < 0.1% at 100 keV)

INTRA-CAVITY LASER INTERFEROMETRY

- This approach relies on changes of the amplitude of the laser emission when the laser light is reflected back to itself by an external mirror.
  - Interference between the cavity oscillations and the reflected beam strongly modulates the laser intensity. Such modulation depends on the phase of the reflected beam.

- Application to plasmas: modulation of the laser intensity is related to the changes of the index of refraction.

\[
N_{[t_1,t_2]} = \frac{2\Delta \mu L}{\lambda_0} \quad \rightarrow \quad \Delta \bar{n}_e = 1.12 \cdot 10^{13} \frac{N_{[t_1,t_2]}}{\lambda_0} \text{[cm}^{-3}\text{]} 
\]
GENERAL CONSIDERATIONS

➤ Spectrally resolved detection systems.

- **Avalanche photo-Diodes.**

  Cascade of interference filters
  3 – 8 wavelength channels
  1 APD for each wavelength

Assume Maxwellian electron distribution!
Spectrally resolved detection systems.
- Intensified image detectors.
- Avalanche photo-Diodes.

Light Detection and Ranging (LIDAR) TS system
- The measurement position can be retrieved from time of flight of the laser pulse.
- The spatial resolution is determined by the physical length of the laser pulse and the temporal response of the detection system.
  - spatial resolution is in the range of 50 - 100 mm
  - Of interest for large fusion devices JET & ITER
REQUIRED LASER POWER AND DETECTION EFFICIENCY

\[ N_{pe} = \frac{E}{h \nu_0} \Delta L \Omega n_e \frac{d\sigma_T}{d\Omega} \tau_{overall} \eta \]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Units/Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy per laser pulse</td>
<td>( E )</td>
<td>J</td>
</tr>
<tr>
<td>f-number of viewing lens</td>
<td>( f/nr )</td>
<td>rad</td>
</tr>
<tr>
<td>Solid angle</td>
<td>( \Omega = \pi (f/nr)/4 )</td>
<td>sr</td>
</tr>
<tr>
<td>Length of the scattering volume</td>
<td>( \Delta L )</td>
<td>m</td>
</tr>
<tr>
<td>Differential cross-section</td>
<td>( d\sigma_T/d\Omega ) = 7.94 \times 10^{-30}</td>
<td>m²/sr</td>
</tr>
<tr>
<td>Overall transmission</td>
<td>( \tau_{overall} )</td>
<td></td>
</tr>
<tr>
<td>Effective quantum efficiency</td>
<td>( \eta )</td>
<td>%</td>
</tr>
<tr>
<td>Electron density</td>
<td>( n_e )</td>
<td>m⁻³</td>
</tr>
</tbody>
</table>
CALIBRATIONS

- Absolute calibration using a known gas pressure
  - This can be done using Raman or Raleigh scattering to determine the ABSOLUTE sensitivity of the detection

\[
N_{pe} = \frac{E}{h\nu_0} \Delta L \Omega \sigma_{\text{Raman or Rayleigh}} \tau_{\text{overall}} \eta
\]